Waves generated by horizontally oscillating bottom disturbances

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Introduction

Moving disturbances on the seafloor are used as wave makers to generate long waves. Past studies on generation of periodic waves by moving disturbances on the seafloor are rare. The problem was investigated by Chen et al. (2022) who considered a vertically oscillating disturbance. In this work, attention is confined to generation of periodic waves by horizontally oscillating disturbances on the seafloor by use of the Level I Green-Naghdi (GN) equations. The GN results are compared with the results of the computational fluid dynamics (CFD) method, and very good agreement is observed. Then a range of variables, including geometrical shape of the disturbance and disturbance oscillations, are considered to assess their effect on the waves generated. It is shown that regular linear and nonlinear waves can be generated by horizontal oscillations of bottom disturbances.

The level I GN Equations

By postulating a kinematic assumption over a deformable fluid sheet, Green & Naghdi (1976) developed the GN equations to study nonlinear wave propagation in an inviscid and incompressible fluid. The GN equations are classified based on the functions used to prescribe the distribution of vertical velocity over the water column. In the Level I GN equations, the vertical velocity field is distributed linearly and the horizontal velocity is constant over the water column. The GN equations of any levels satisfy the nonlinear boundary conditions and conservation of mass and postulates conservation of momentum in an integrated manner. There is no further kinematic limitation for the flow field in the theory i.e. the fluid flow can be rotational.

In this study, a deformable fluid sheet is established in two dimensions. The right-hand side Cartesian coordinate system is used, where x_1 points towards the right and x_2 points upwards, against the gravity. Ertekin (1984) provided a compact form of the Level I GN governing equations as

$$\eta_{,t} + \{(h+\eta-\alpha)u_1\}_{,x_1} = \alpha_{,t}, \tag{1}$$

$$\dot{u}_1 + g\eta_{,x_1} + \frac{\hat{p}_{,x_1}}{\rho} = -\frac{1}{6}\{[2\eta+\alpha]_{,x_1}\ddot{\alpha} + [4\eta-\alpha]_{,x_1}\ddot{\eta} + (h+\eta-\alpha)[\ddot{\alpha}+2\ddot{\eta}]_{,x_1}\}, \tag{2}$$

$$u_2(x_1,x_2,t) = \dot{\alpha} + \frac{x_2+h-\alpha}{h+\eta-\alpha}(\dot{\eta}-\dot{\alpha}), \tag{3}$$

where η is the surface elevation, measured from the still-water level (SWL), h is the constant water depth, α is the bottom deformation, u_1 and u_2 are the horizontal and vertical velocities, respectively, g is the gravity acceleration, ρ is the water density, $\hat{p}$ is the pressure on the top surface of the fluid sheet. The subscripts after comma indicates partial differentiation with respect to the corresponding variables. For an arbitrary variable θ, $\dot{\theta}$ and $\ddot{\theta}$, represent the first-order and second-order material derivatives, respectively. All variables in this study are dimensionless by using ρ, g and h, as a dimensionally independent set, and thus we write

$$x_1' = \frac{x_1}{h}, \quad \eta' = \frac{\eta}{h}, \quad \alpha' = \frac{\alpha}{h}, \quad u_1' = \frac{u_1}{\sqrt{gh}}, \quad t' = t\sqrt{\frac{g}{h}}. \tag{4}$$

The superscript ($'$) are removed from all variables for simplicity.

Horizontally oscillating bottom disturbance

In this study, the moving bottom disturbance oscillates harmonically in the horizontal direction. Hence, attention is confined to the continuous moving disturbance on the seafloor, as shown in Fig. 1. The bottom disturbance, $\alpha(x_1,t)$, is an arbitrary continuous function of x_1 and t. In this study, we use the following function to describe the deformation of the seafloor, although this can be any continuous function in general,

$$\alpha(x_1,t) = A_0 \times R(x_1,t) \times D(x_1,t)\,, \tag{5}$$

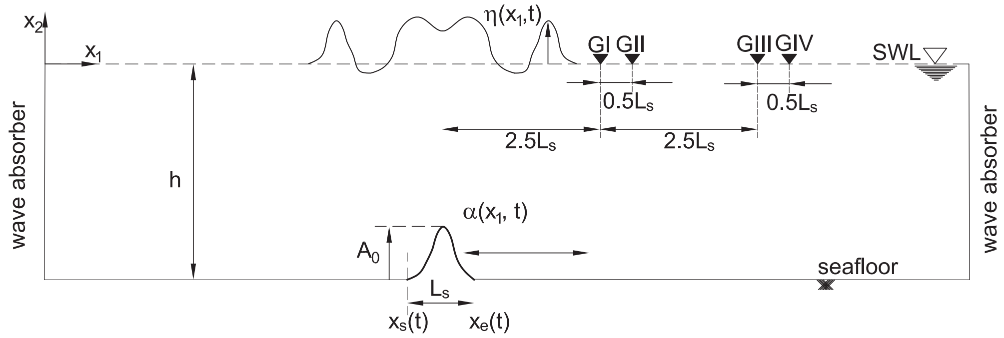

Figure 1: The sketch of the problem of waves generated by an arbitrary-shaped continuous bottom disturbance, $\alpha(x_1, t)$. x_s and x_e are the starting and ending positions of the bottom disturbance, and $L_s = x_e - x_s$ is the disturbance length, and A_0 is the constant disturbance amplitude.

where A_0 is the amplitude of the bottom disturbance, $D(x_1, t) = sech^2(x_1 - x_0)$, where $x_0(t) = X_0 + A_h sin(\omega t)$ is the instantaneous horizontal location of the center of the disturbance, and X_0 is the initial location of the disturbance center. A_h and ω are the oscillation amplitude (in the horizontal direction) and oscillation frequency of the disturbance, respectively. $R(x_1, t) = e^{\frac{-(x_1-x_0)^2}{\sigma^2}}$ is the ramp function, where $\sigma = \frac{L_s}{4}$ is constant, controlling the effective length of $R(x_1, t)$.

Due to the moving bottom, spatial and time derivatives of α are specified and substituted into Eqs.(1) and (2)

$$\begin{aligned}
&\alpha_{,t} = A_0 \times (R_{,t}D + RD_{,t}), \quad \alpha_{,tt} = A_0 \times (R_{,tt}D + 2R_{,t}D_{,t} + RD_{,tt}) \\
&\alpha_{,x_1t} = A_0 \times (R_{,x_1t}D + R_{,t}D_{,x_1} + R, x_1D_{,t} + RD_{,x_1t}), \\
&\alpha_{,x_1x_1t} = A_0 \times (R_{,x_1x_1t}D + 2R_{,x_1t}D_{,x_1} + 2R_{,x_1}D_{,x_1t} + R_{,t}D_{,x_1x_1} + R_{,x_1x_1}D_{,t} + RD_{,x_1x_1t}), \\
&\alpha_{,x_1tt} = A_0 \times (R_{,x_1tt}D + 2R_{,x_1t}D_{,t} + 2R_{,t}D_{,x_1t} + R_{,tt}D_{,x_1} + R_{,x_1}D_{,tt} + RD_{,x_1tt}), \\
&\alpha_{,x_1} = A_0 \times (R_{,x_1}D + RD_{,x_1}), \quad \alpha_{,x_1x_1} = A_0 * (R_{,x_1x_1}D + 2R_{,x_1}D_{,x_1} + RD_{,x_1x_1}), \\
&\alpha_{,x_1x_1x_1} = A_0 \times (R_{,x_1x_1x_1}D + 3R_{,x_1x_1}D_{,x_1} + 3R_{,x_1}D_{,x_1x_1} + RD_{,x_1x_1x_1}).
\end{aligned} \tag{6}$$

The term $\alpha_{,t}$ contributes to the right hand side of Eq. (1) while the others terms are expanded by Eq. (2), see Chen et al. (2022).

The domain of the deformable fluid sheet is discretized into a set of mesh points and all continuous variables are approximated by the discrete values on the mesh by using the finite difference method. The spatial derivatives are determined by use of the second-order central difference method and the Modified Euler Method is used for time marching.

Results and Discussion

Surface elevation time series, recorded at gauges GI, GII, GIII and GIV, whose locations are shown in Fig. 1, are given in Fig. 2. Results are compared with those obtained by use of the CFD results obtained by use of OpenFOAM. It is observed that clean regular waves are generated by the horizontally oscillating bottom disturbance, and the waves keep their forms as they propagate along the tank. Overall, results calculated by the GN model are close to those of the CFD method.

Snapshots of bottom disturbances and wave profiles at different times for different A_0 values are shown in Fig. 3. Also, Fig. 4 shows the time series of surface elevation for a range of A_0 values. Shown in Figs. 3 and 4, amplitudes of the generated waves appear to be linearly proportional to A_0.

Time series of surface elevation for different Ls values are shown in Fig. 5. It is observed that wave amplitudes vary nonlinearly with L_s. Wave amplitudes generally increase with longer L_s. However, we find that wave amplitudes do not change remarkably with increasing L_s, when $L_s \geq 8$.

Time series of surface elevation for various A_h values are shown in Fig. 6. We see that wave amplitudes vary nonlinearly with the oscillation amplitude A_h. Wave amplitudes generated by the disturbance increase with the increasing oscillation amplitude, but dependence of the wave amplitudes on A_h becomes insignificant when $A_h \geq 0.8$.

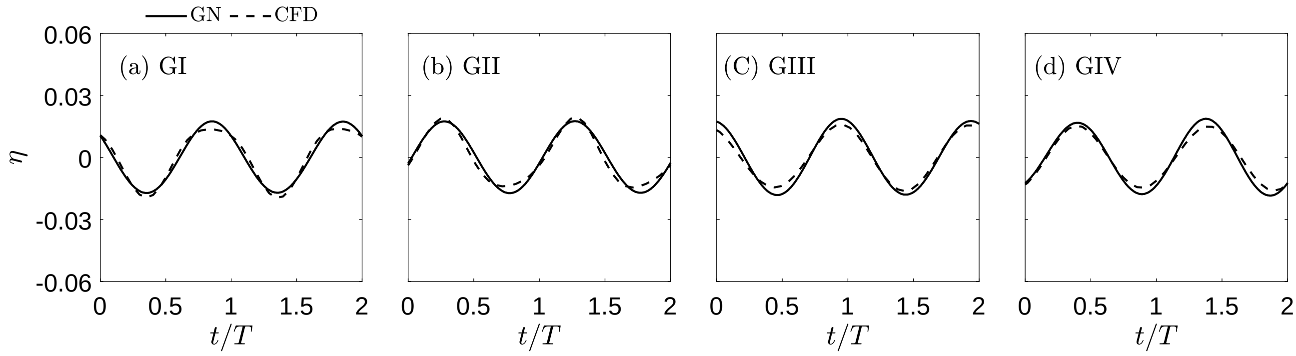

Figure 2: Comparisons of surface elevation recorded at gauges GI, GII, GIII and GIV, by the GN and CFD approaches. $L_s = 4$, $A_0 = 0.1$, $A_h = 0.8$, $T = 6$, and $T = \frac{2\pi}{\omega}$ is the oscillation period.

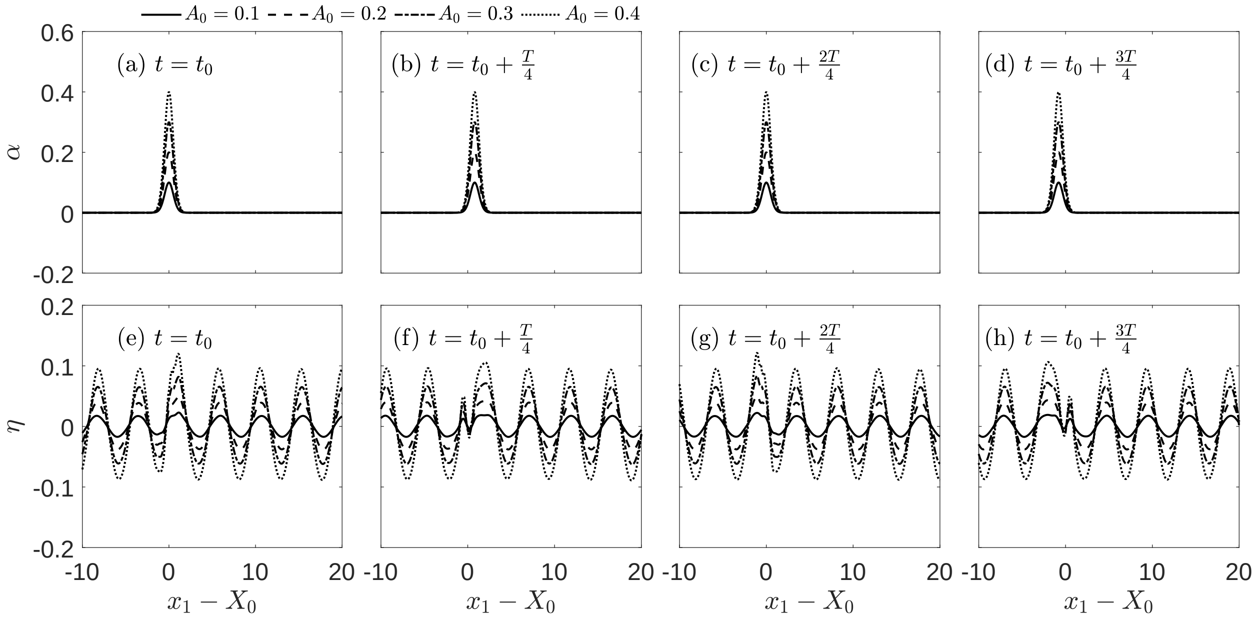

Figure 3: Snapshots of (a-d) bottom disturbances and (e-h) wave profiles at $t = t_0$, $t = t_0 + \frac{T}{4}$, $t = t_0 + \frac{2T}{4}$, and , $t = t_0 + \frac{3T}{4}$, for various geometry amplitudes A_0. $L_s = 4$, $A_h = 0.8$, $T = 6$.

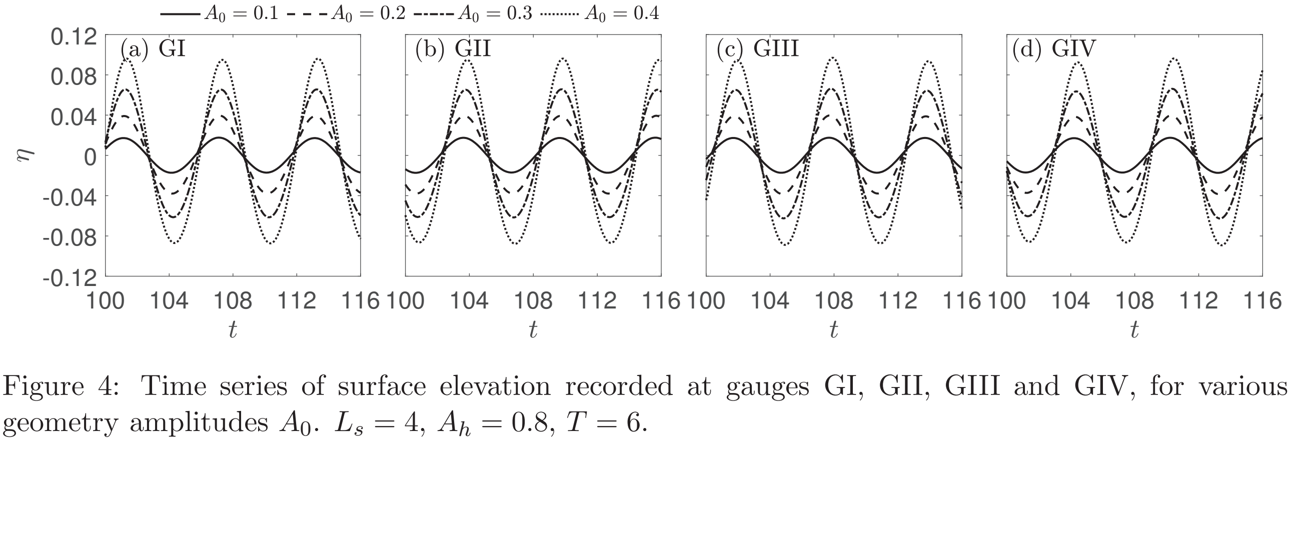

Figure 4: Time series of surface elevation recorded at gauges GI, GII, GIII and GIV, for various geometry amplitudes A_0. $L_s = 4$, $A_h = 0.8$, $T = 6$.

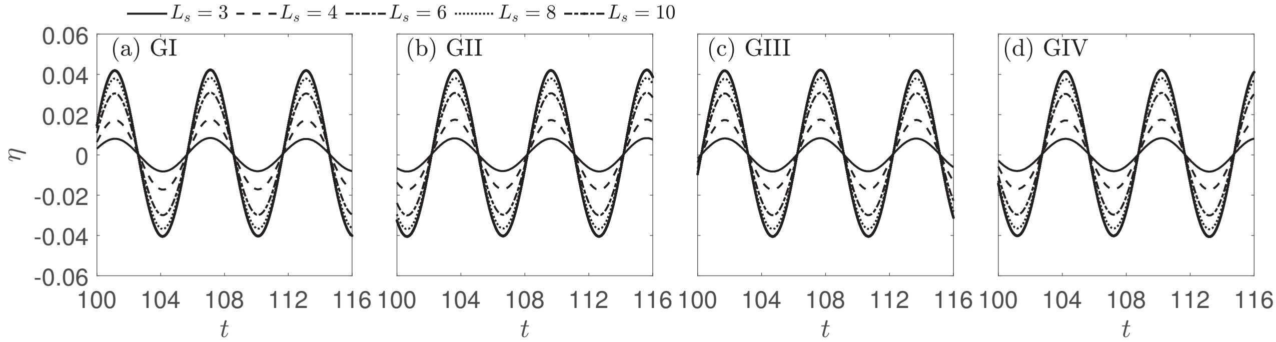

Figure 5: Time series of surface elevation recorded at gauges GI, GII, GIII and GIV, for various geometry length L_s. $A_0 = 0.1$, $A_h = 0.8$, $T = 6$.

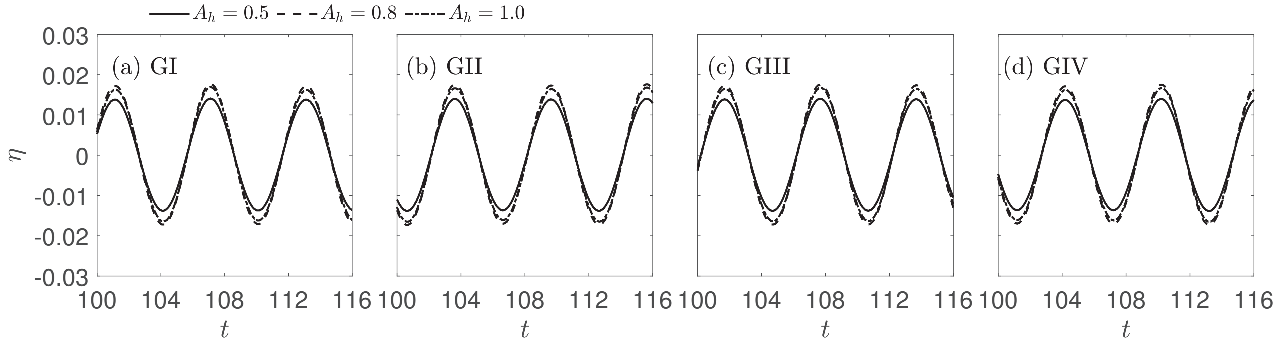

Figure 6: Time series of surface elevation recorded at gauges GI, GII, GIII and GIV, for a variety of oscillation amplitudes A_h. $L_s = 4$, $A_0 = 0.1$, $T = 6$.

Time series of surface elevation for different T values are shown in Fig. 7. It is shown that wave amplitudes are nearly invariant with T. Wave peak values of $T = 6$ are slightly larger than that of $T = 5$ and $T = 7$ while wave trough values of $T = 6$ are much closer to the other two cases.

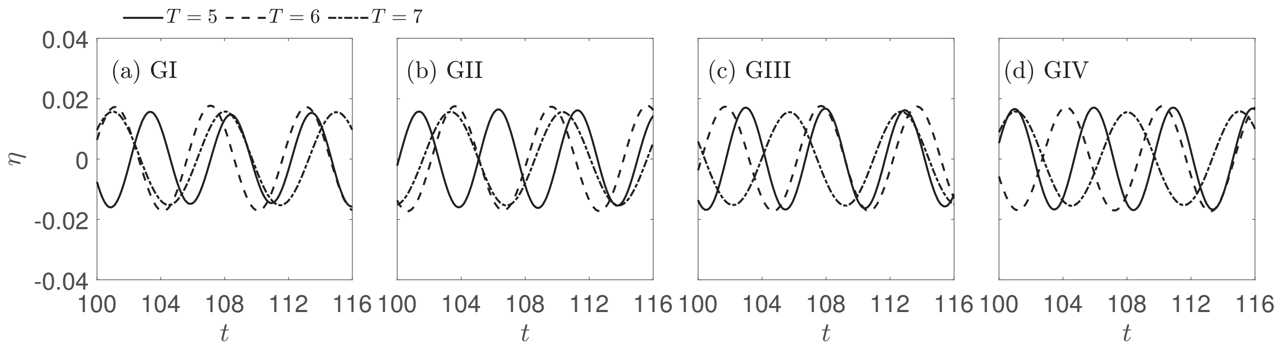

Figure 7: Time series of surface elevation recorded at gauges GI, GII, GIII and GIV, for a variety of oscillation periods T. $L_s = 4$, $A_0 = 0.1$, $A_h = 0.8$.

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