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Vertical Oscillation of a Submerged Horizontal Plate

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Highlights

- Interaction of nonlinear waves with an oscillating, horizontal, submerged plate is studied by use of the Level I Green-Naghdi equations.
- Vertical oscillation of the plate is determined by solving the equation of motion, subject to the time- and spatial-varying wave-induced forces, and damping and spring external forces.

Introduction

Many wave energy converter devices consist of an oscillatory body that is connected to an electricity generator. Survivability of the oscillating devices, located on or close to the surface, has been a major difficulty in design of wave energy converters. In addition, the devices are usually located in shallow to intermediate waters, where the interaction of nonlinear shallow water waves with an oscillating object has been a challenging objective for research and development. Here, we propose the use of an oscillatory, horizontal and fully submerged plate, located in shallow water, as the core component of a wave energy converter.

The vertical motion of a fully submerged plate due to nonlinear waves in shallow water is studied here. The nonlinear wave-induced horizontal and vertical forces on the submerged, oscillating plate vary with time and the instantaneous submergence depth, and they are calculated by use of the Level I Green-Naghdi (GN hereafter) equations. Attention is confined to the nonlinear motion of the submerged plate due to waves and other external forces. Effect of the power-takeoff system is simplified by spring and damping forces on the oscillatory plate.

The Green-Naghdi Equations

The GN equations for propagation of nonlinear waves were originally developed based on the theory of directed fluid sheets of shells and plates by Green & Naghdi (1974). In this theory, the continuum fluid is assumed to be incompressible; viscosity is not a constraint in the general form of the theory. No assumption of irrotationality of the flow is made. The GN equations satisfy the free-surface and seafloor boundary conditions, and the integrated conservation laws of mass and momentum (and energy in general), exactly. In water wave applications, the theory may be classified by the *Levels*, prescribing variation of the vertical component of velocity along the water column.

In this study, the Level I GN equations are considered in their two-dimensional form. We use a rectangular Cartesian coordinate system (x_1, x_2), where the origin of the coordinate system is located on the still-water level (SWL), with x_1 pointing to the right, and x_2 pointing upward against gravity. The free surface, $\eta(x_1, t)$, is measured from the SWL, and h is the water depth, and it is assumed constant here, though such assumption is not a general requirement. In the Level I GN equations, variation of the vertical component of velocity, u_2, is assumed to be linear over the water column, i.e. $u_2(x_1, x_2, t) = C.(x_2 + h)$, where $C = C(x_1, t)$. This is the only assumption made about the dynamic of the fluid sheet, and along with the incompressibility condition, it results in the horizontal component of velocity, u_1, to be constant over the water column, i.e. $u_1(x_1, x_2, t) = u_1(x_1, t)$ in two-dimensions. Such an assumption, made in the Level I equations, is mostly applicable to propagation of fairly long water waves. Further details about the Level I GN equations can be found in e.g., Ertekin et al. (1986). Applications of higher levels GN equations to some ocean and coastal engineering problems can be found in, e.g. Ertekin et al. (2014) and Zhao et al. (2014).

The final form of the Level I GN equations for propagation of waves in an incompressible and inviscid fluid over a stationary floor are given by the mass and combined momentum equations:

$$\eta_{,t} + \{(h+\eta-\alpha)u_1\}_{,x_1} = 0\,, \tag{1a}$$

$$\dot{u}_1 + g\eta_{,x_1} + \frac{\hat{p}_{,x_1}}{\rho} = -\frac{1}{6}\{[2\eta+\alpha]_{,x_1}\ddot{\alpha} + [4\eta-\alpha]_{,x_1}\ddot{\eta} + (h+\eta-\alpha)[\ddot{\alpha}+2\ddot{\eta}]_{,x_1}\}\,, \tag{1b}$$

where $\hat{p}(x_1,t)$ is the pressure on the top surface of the fluid sheet, $\alpha(x_1)$ is the bottom surface of the fluid sheet, ρ is the fluid density and g is the gravitational acceleration. Superposed dots in Eq. (1) denote the two-dimensional material time derivative and double dots are defined as the second material time derivative. All lower case latin subscripts after commas in Eq. (1) designate partial differentiation with respect to the indicated variables.

Recently, Hayatdavoodi & Ertekin (2012) and Hayatdavoodi & Ertekin (2015*a*) obtained the wave-induced loads on a fixed, submerged, horizontal plate by solving the Level I GN equations. In this approach, the domain is discretized into four regions, namely upwave (I), above the plate (II), below the plate (III), and downwave (IV). Appropriate equations in each region is then solved for the unknowns: u_1 and η in Regions I, II and IV, where the top surface is free and the top pressure is equal to the atmospheric pressure, and u_1 and $\hat{p}$ in Region III, under the plate, where the top surface of the fluid sheet is fixed (the plate bottom). Results of this model were compared with CFD computations and laboratory experiments, and overall close agreement is observed, see e.g. Hayatdavoodi & Ertekin (2015*b*).

Oscillation of a Submerged Plate

The two-dimensional Level I GN equations are used to determine the vertical oscillation of the submerged, horizontal plate. The plate is thin and rigid, and it remains horizontal and submerged at all times. The plate may only oscillate in the vertical, x_2, direction; it is restricted from any motion in the horizontal direction. This may be achieved by using vertical, thin guide rails for example. Similar approach as that used by Hayatdavoodi & Ertekin (2015*a*) is followed here to analyze the interaction of nonlinear waves with the oscillating plate.

In the case of the oscillating plate, the thickness of the fluid sheets in Regions II and III, above and below the plate, are variable in time, i.e. $h_{II} = h_{II}(t)$ and $h_{III} = h_{III}(t)$. Consequently, the governing equations are to be calculated for the instantaneous thickness of the fluid sheet. In Regions I, II and IV, the equations of motion of the fluid, Eq. (1), and pressures in the fluid sheet are given by

$$\eta_{,t} + \{(h+\eta)u_1\}_{,x_1} = 0\,, \quad \dot{u}_1 + g\eta_{,x_1} = -\frac{1}{3}\{(2\eta_{,x_1}\ddot{\eta}) + (h+\eta)\,\ddot{\eta}_{,x_1}\}\,, \tag{2a}$$

$$P = \left(\frac{\rho}{6}\right)(h+\eta)^2\,(2\ddot{\eta}+3g)\;, \quad \bar{p} = \left(\frac{\rho}{2}\right)(h+\eta)\,(\ddot{\eta}+2g)\;, \tag{2b}$$

where $h = h_I$ in Regions I and IV, and it is constant, and $h = h_{II}\,(t)$ in Region II, $P(x_1,t)$ is the integrated pressure over x_2, and $\bar{p}$ is the pressure of the bottom surface. In Region III, where $\eta = 0$ at all times, the equations of motion of the fluid and pressures are given by

$$\{u_1 h_{III}\}_{,x_1} = 0\,, \quad \dot{u}_1 + \frac{\hat{p}_{x_1}}{\rho} = 0\,, \quad P = \frac{1}{2}\left(\rho g h_{III}^2\right) + \hat{p}h_{III}\,, \quad \bar{p} = (\rho g h_{III}) + \hat{p}\,. \tag{3}$$

To obtain a continuous solution in the entire domain, the jump and matching conditions are used at the leading and trailing edges of the oscillating plate. The jump conditions, demanded by the theory, ensure conservation of mass and momentum across the discontinuity curve, and the matching conditions, demanded by the physics of the problem, ensure continuous surface elevation and bottom pressure throughout the domain.

The equation of vertical motion of the plate is given by Newton's second law: $\Sigma F = ms_{,tt}$, where m is the mass of the two-dimensional plate (mass per unit length of the plate), $s = s(t) = h_{II}(t)$ is the instantaneous submergence depth measured from the SWL to the plate, $s_{,tt}$ is the instantaneous acceleration of the plate, and ΣF is the sum of all the forces acting on the plate in the vertical direction. Here, we assume that ΣF consists of the following forces:

$$\Sigma F = F_{x_2} + F_{st} + F_f + F_{PT}\,, \tag{4}$$

where $F_{x_2}(x_2, t)$ is the instantaneous wave-induced vertical force on the plate, calculated by the GN equations, F_{st} is the constant hydrostatic force on the plate defined as the difference between the buoyancy force and weight of the plate, $F_f(x_2, t)$ is the frictional force due to the motion of the plate on the guide rails, and F_{PT} is the time- and spatial-varying force exerted by the power-takeoff system on the plate. The frictional force is given by $F_f(x_2, t) = -\left(\frac{|s_{,t}|}{s_{,t}}\right)\mu F_{x_1}$, where μ is the constant coefficient of friction, $s_{,t}$ is the instantaneous velocity of the plate, and $F_{x_1}(x_2, t)$ is the instantaneous horizontal force on the plate calculated by the GN equation. The power-takeoff force is assumed to consist of two components: $F_{PT}(x_2, t) = -k\,(s - s_0) - cs_{,t}$, where s_0 is the initial submergence depth, and k and c are the spring stiffness and viscous damping coefficient, respectively, i.e., we assume that external spring and damping forces work on the plate due to the power-takeoff system and the control mechanism. To prevent the plate from approaching the free surface or seafloor, constraints are set to limit the range of oscillation, although this is not required in general.

The coupled system of equation of fluid motion and plate oscillation are solved simultaneously in the entire domain. The system of equations of the fluid is solved numerically by the central-difference method, second-order in space, and with the Modified-Euler Method for time marching. The plate equation of vertical motion, $\Sigma F = ms_{,tt}$, is solved by use of a 4th-order Runge-Kutta method, assuming an initial condition for all variables.

Results

Preliminary results of the oscillation of a submerged plate (plate width $B/h_I = 15$ and initial submergence $s_0/h_I = 0.5$) due to incoming cnoidal waves (wavelength $\lambda/h_I = 20$ and wave height $H/h_I = 0.2$) are given here. Figure 1 presents the time history of plate vertical oscillation ($\bar{s} = s/h_I$), where plate mass $m/\left(\rho h^3\right) = 1$, and $k = 15$, $c = 5$ and $\mu = 0.4$, all dimensionless coefficients, are considered here. Surface elevation $\bar{\eta} = \eta/h_I$, recorded at a wave gauge located in the middle of the plate is also shown in this figure. The period of plate oscillation is the same as the incoming wave, however, the plate oscillation includes some higher harmonics. In this case, the plate oscillates between $\bar{s} = 0.4$ and $\bar{s} = 0.55$. A comparison of time history of plate vertical oscillation for different cnoidal wave heights is shown in Fig. 1. It is shown that the crest of the nonlinear plate oscillation increases with larger wave height, while the trough remains the same, i.e., plate oscillates closer to the surface with larger wave heights.

Figure 2 shows a comparison of the surface elevation, and the vertical and horizontal forces on a fixed submerged plate and on an oscillating plate under the same conditions. Oscillation of the plate modifies the wave field, and may result in formation of small amplitude radiating waves, seen as higher harmonics in the surface elevation. In this case, the wave-induced forces on the submerged plate does not show significant changes due to the plate oscillation.

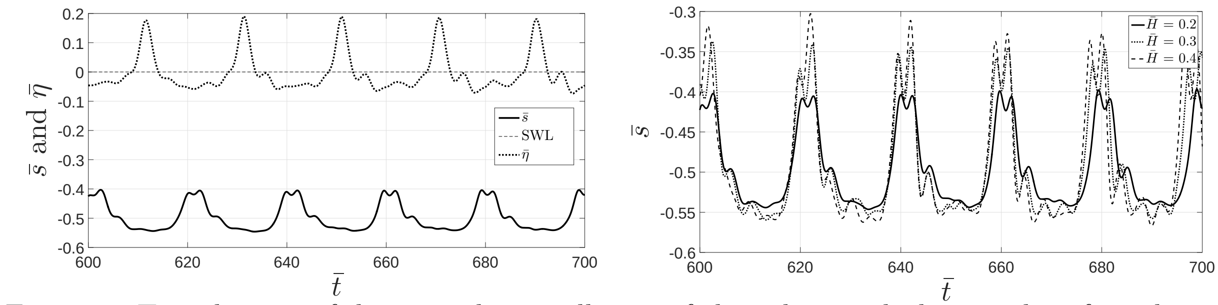

Figure 1: Time history of dimensionless oscillation of the submerged plate, and surface elevation above the plate (left), and a comparison of plate oscillation for different wave heights (right).

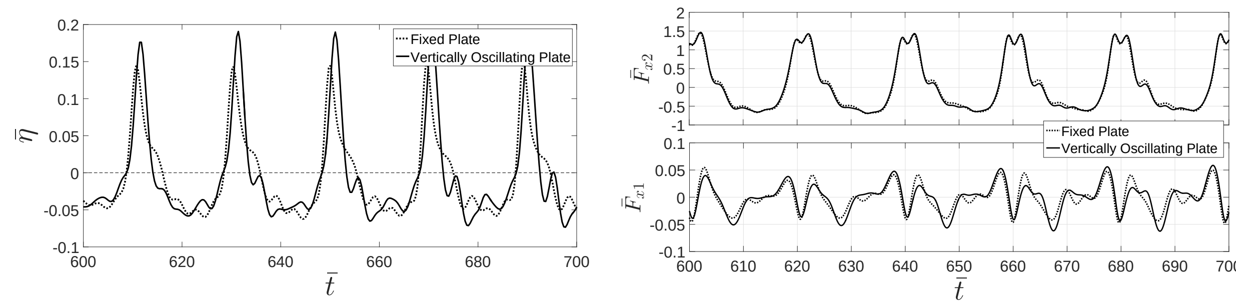

Figure 2: Comparison of surface elevation above the plate recorded by a gauge located in the middle of the plate (left), and the cnoidal wave force (right), for a fixed and oscillating plate. The dimensionless forces are given as $\bar{F}_{x_1} = F_{x_1}/\rho g h_I t_p L_p$ and $\bar{F}_{x_2} = F_{x_2}/\rho g h_I^2 L_p$, where L_p and t_p are the plate width and plate thickness, respectively. The fixed plate is located at $\bar{s} = 0.5$.

Conclusions

The nonlinear vertical oscillation of a submerged, horizontal plate is studied by the Level I GN equations. The plate oscillation follows the wave period, and the height and nonlinearity of the oscillations can be controlled by external damping and spring effects. These are promising results for application of the oscillatory plate in a wave power extraction device, where the plate will be attached to a direct drive generator. Preliminary results of CFD computations and laboratory experiments show good agreement with the GN equations; these will be discussed later.

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