TEXAS A&M UNIVERSITY
GALVESTON CAMPUS®

Nonlinear Wave Scattering by a Submerged Horizontal Plate

2016 Ocean Sciences Meeting, 21-26 February, New Orleans, LA, U.S.A

Benjamin Valentine[1], Masoud Hayatdavoodi[1], R. Cengiz Ertekin[2]

1.Dept. of Ocean Engineering, Texas A&M University, Galveston, Texas 77554, U.S.A.: 2. Ocean and Resources Engineering Dept., University of Hawaii at Manoa, Honolulu, HI 96822, U.S.A.

1. INTRODUCTION

Breakwaters are designed to protect coastal infrastructure by dissipating wave energy, partially due to wave reflection. A submerged, horizontal deck located beneath free surface can be used as a breakwater. Some benefits include:

- Less dependent on seafloor topography for foundation
- No visual distractions, nor limits recreational activities in coastal regions,
- Does not restrict natural circulation of water and sediment transport,
- It is a simple geometry, and easy to construct and maintain

Submerged plates act as breakwaters by reflecting part of the wave due to the fluid-structure and fluid-fluid interaction, while the remaining wave energy is transferred above and below the plate.

Many studies have been conducted on a submerged plate using linear deep water or intermediate water waves. We will study the effects of a submerged plate as a breakwater using the Green-Naghdi (GN) nonlinear, shallow water wave equation.

2. SUBMERGED BREAKWATER

Figure 1 shows a schematic of submerged horizontal plate as a breakwater. B is plate length, h is water depth, h_{II} is submergence depth from the still water level (SWL), h_{III} is the plate vertical location from the seafloor, $\eta_I + \eta_R$ is the combined reflected and incident waves upwave of the plate, and η_T is the transmitted waves downwave.

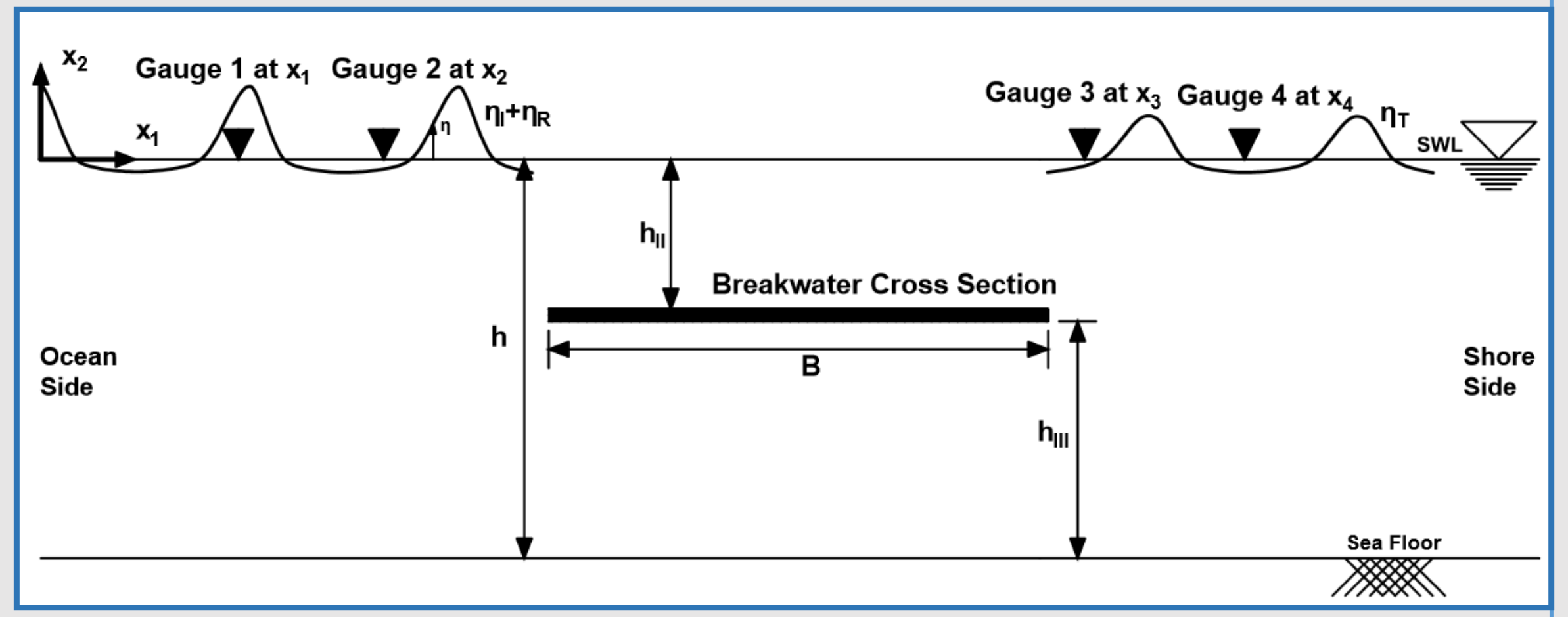

Fig. 1: Schematic of submerged horizontal plate as a breakwater.

3. METHODOLOGY

The Level I Green-Naghdi (GN) equations originally developed by Green and Naghdi (1976a, 1976b) are used to solve the propagation of an inviscid and incompressible fluid over a submerged plate. The GN equations are given by the mass and combined momentum equations, Ertekin (1984):

$$\eta_t + [(h+\eta-\alpha)u_1]_{x1} = \alpha \ , \qquad (1)$$

$$\dot{u}_1 + g\eta_{x1} + \frac{\hat{p}_{x1}}{\rho} = -\frac{1}{6}\left\{[2\eta+\alpha]_{x1}\ddot{\alpha} + [4\eta-\alpha]_{x1}\ddot{\eta} + (h+\eta-\alpha)[\ddot{\alpha}+2\ddot{\eta}]_{x1}\right\}, \qquad (2)$$

where $\alpha(x_1,t)$ is the vertical location of the bottom of the fluid sheet, $\hat{p}\,(x_1,t)$ is the free surface pressure, ρ is the fluid's mass density, the superimposed dot is the two-dimensional material time derivative, and g is gravitational acceleration. See Hayatdavoodi and Ertekin (2015) for details on applying the GN equations to the problem of flow of an inviscid and incompressible fluid over a submerged plate.

We measure the effectiveness of a submerged plate as a breakwater using reflection and transmission coefficients to describe wave scattering. A method similar to the two-point method for nonlinear waves given by Grue (1992) is applied to closely estimate scattered wave amplitudes. In this method, we use the dispersion relation of the linearized GN solution, given by Green and Naghdi (1974) as:

$$\omega^2 = \frac{ghK^2}{1+\frac{1}{3}h^2K^2} \ , \qquad (3)$$

where ω is the angular wave frequency, and K is the wave number.

We discern the reflected wave amplitude, a_R, incident wave amplitude, a_I, and transmitted wave amplitude, a_T, by performing a Fourier transform on the surface elevation time series recorded at the wave gauges and performing the method and analysis given by Grue (1992).

The reflection coefficient, C_R, is the ratio of a_R to a_I, given by:

$$C_R = \frac{a_R}{a_I} \ . \qquad (4)$$

The transmission coefficient, C_T, is the ratio of a_T to a_I, given by:

$$C_T = \frac{a_T}{a_I} \ . \qquad (5)$$

Further details obtaining C_R and C_T can be found in Grue (1992).

4. RESULTS AND DISCUSSION

a) Snapshot of the Numerical Wave Tank

In Fig. 2, a snapshot of the GN wave tank with a submerged plate is shown. The vertical axis is the dimensionless surface elevation (η/h), and the horizontal axis is the dimensionless position (X/h). Cnoidal waves travelling from the left to right hand side of the figure are seen scattering can be seen occurring above the submerged plate, resulting in the combined incident and reflected waves travelling upwave from the plate, and transmitted waves travelling downwave from the plate. It is clear that the transmitted wave amplitude is considerably less than the combined incident and reflected wave amplitude.

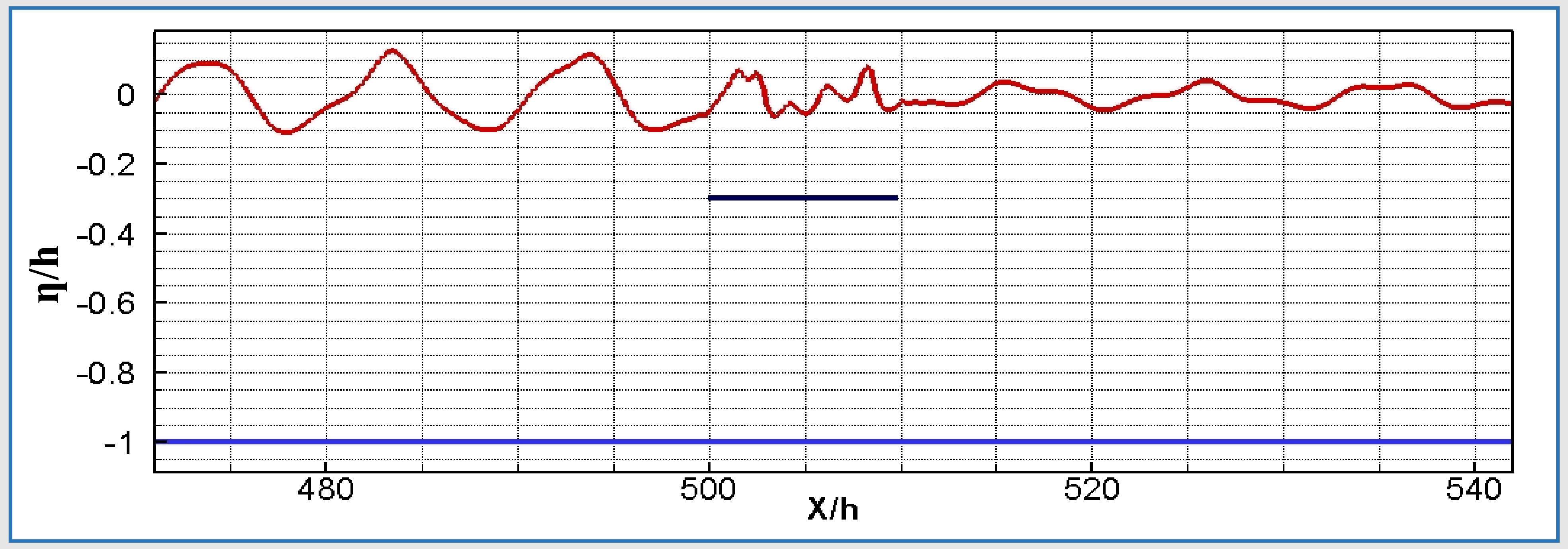

Fig 2: Conditions are H/h = 0.2; B/h = 10; h_{II}/h = 0.3; and L/h = 10. H and L are wave height and wave length, respectively.

b) Surface Elevation Comparisons

Here, the surface elevations (η) calculated by the GN equations are compared with existing data. Figure 3 shows GN comparisons with Computational Fluid Dynamics (CFD) and wave flume measurements given by Hayatdavoodi et al. (2015). Figure 4 shows GN comparisons with the Desingularized Boundary Integral Equation Method (DBIEM) and measurements given by Liu (2009), and a curve fit to model the GN surface elevation recordings for use in Grue's (1992) method. The results of GN compare well.

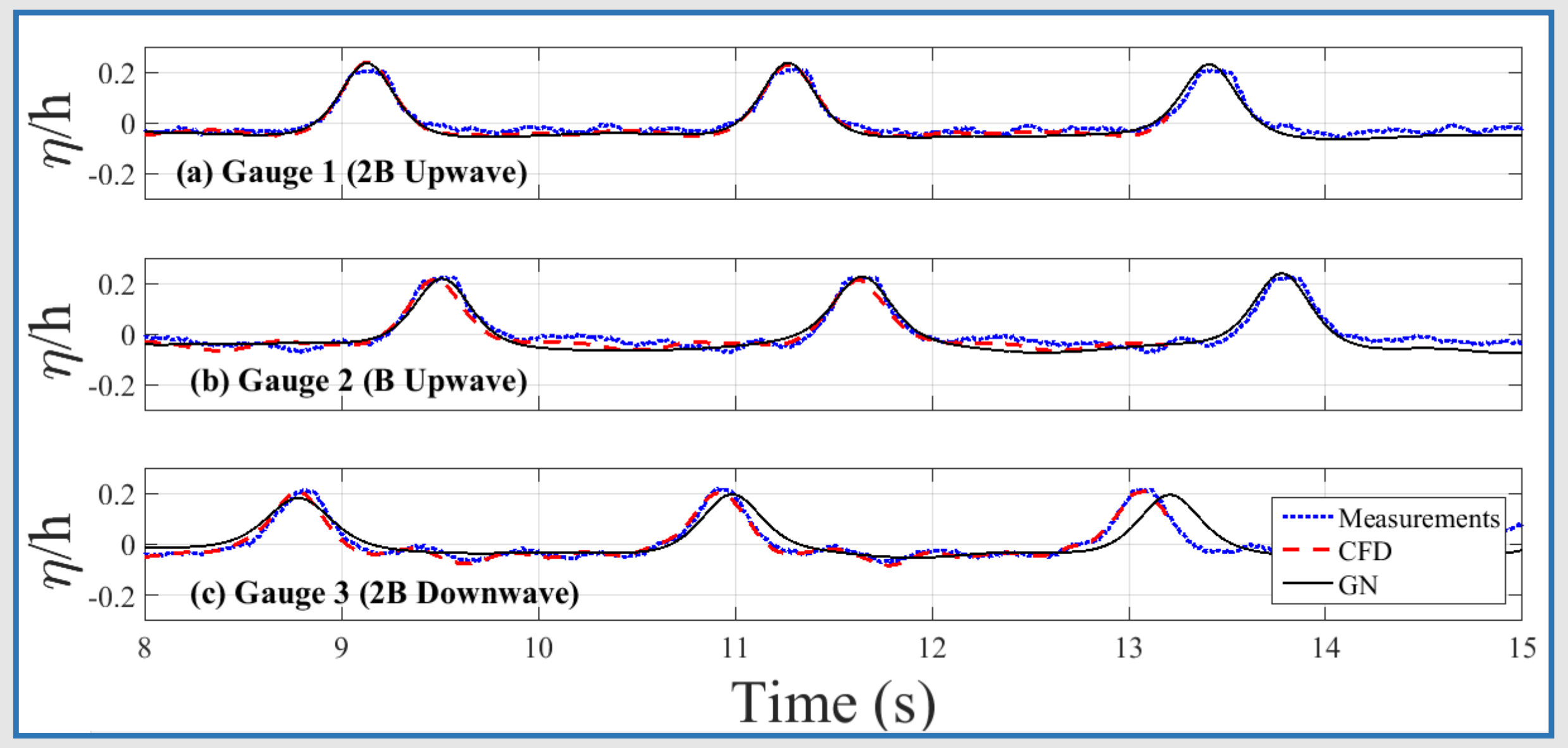

Fig. 3: GN comparison of dimensionless surface elevation (η/h) of waves travelling to the right, given by Hayatdavoodi et al. (2015) recorded at three gauges, Gauge 1 (shown in graph (a)), Gauge 2 (shown in graph (b)) and Gauge 3 (shown in graph (c)). Conditions are H/h = 0.3; h_{II}/h = 0.6; B/h = 2.102; L/B = 6.234.

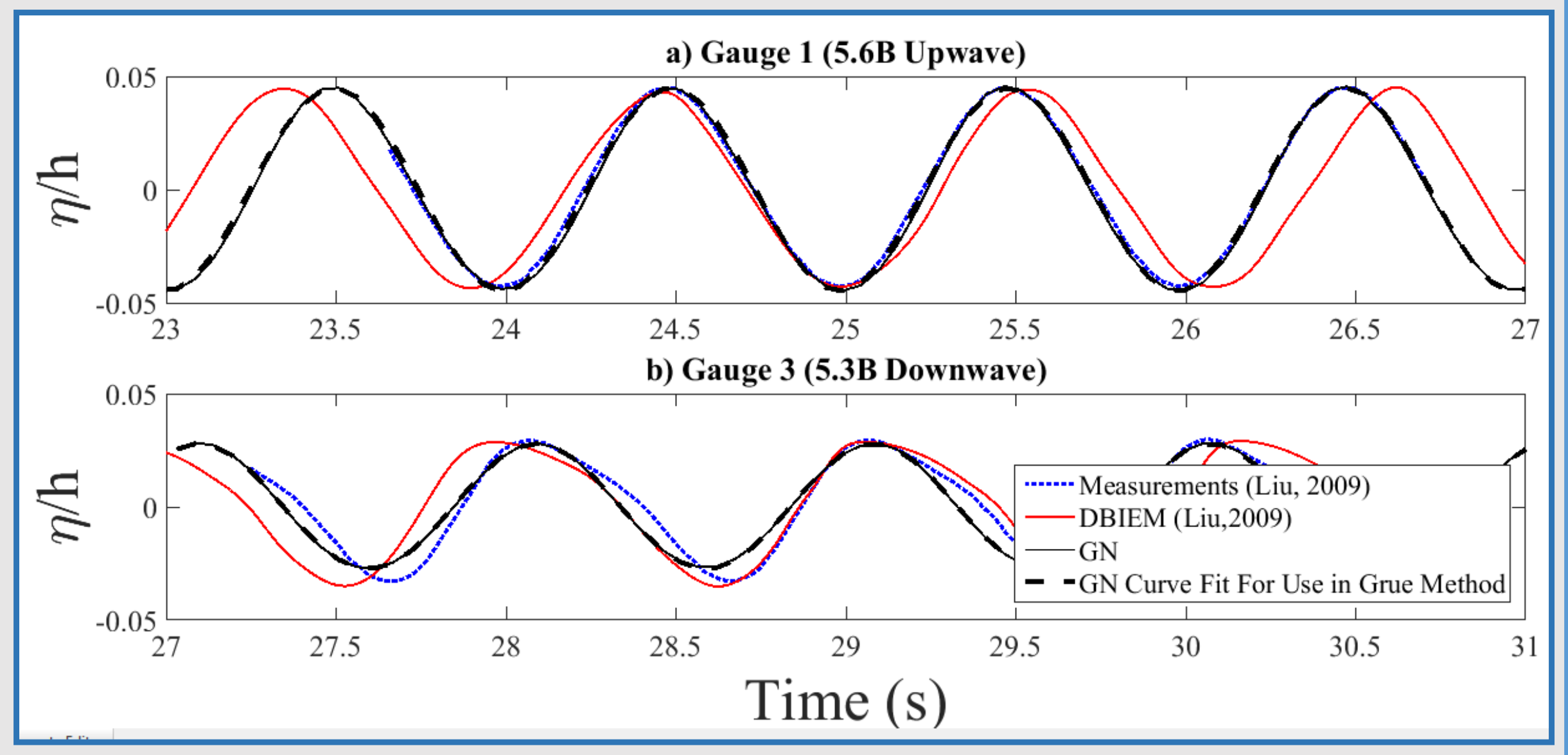

Fig. 4: GN comparison of dimensionless surface elevation (η/h) recorded at two wave gauges, Gauge 1 (shown in graph (a)) and at Gauge 3 (shown in graph (b)). Conditions are H/h = 0.0667; h_{II}/h = 0.333; B/h = 2.0; L/B = 2.524.

c) Reflection and Transmission Coefficient Comparisons

The reflection coefficient, C_R, and transmission coefficient, C_T, determined from the GN equation surface elevation time series are compared with measurements from Brossard (2009), Liu (2009) and DBIEM of Liu (2009), and Lin (2014) Boundary Element Method (BEM). Plate submergence depth, h_{II}/h is varied in Fig 5; wave length to plate length ratio, L/B, is varied in Fig. 6. GN shows good comparison. It is observed within the studies shown that reflection coefficient increases as submergence depth decreases. The relation, however, remains nonlinear and as the submergence depth decreases, smaller variation is observed. As expected, C_T shows opposite behavior compared to C_R. As submergence depth decreases, C_T also decreases, and C_T increases as the submergence depth increases. For decreasing L/B ratios less than 3, C_R decreases while C_T increases. For increasing L/B ratios larger than 3, C_R again decreases while C_T increases.

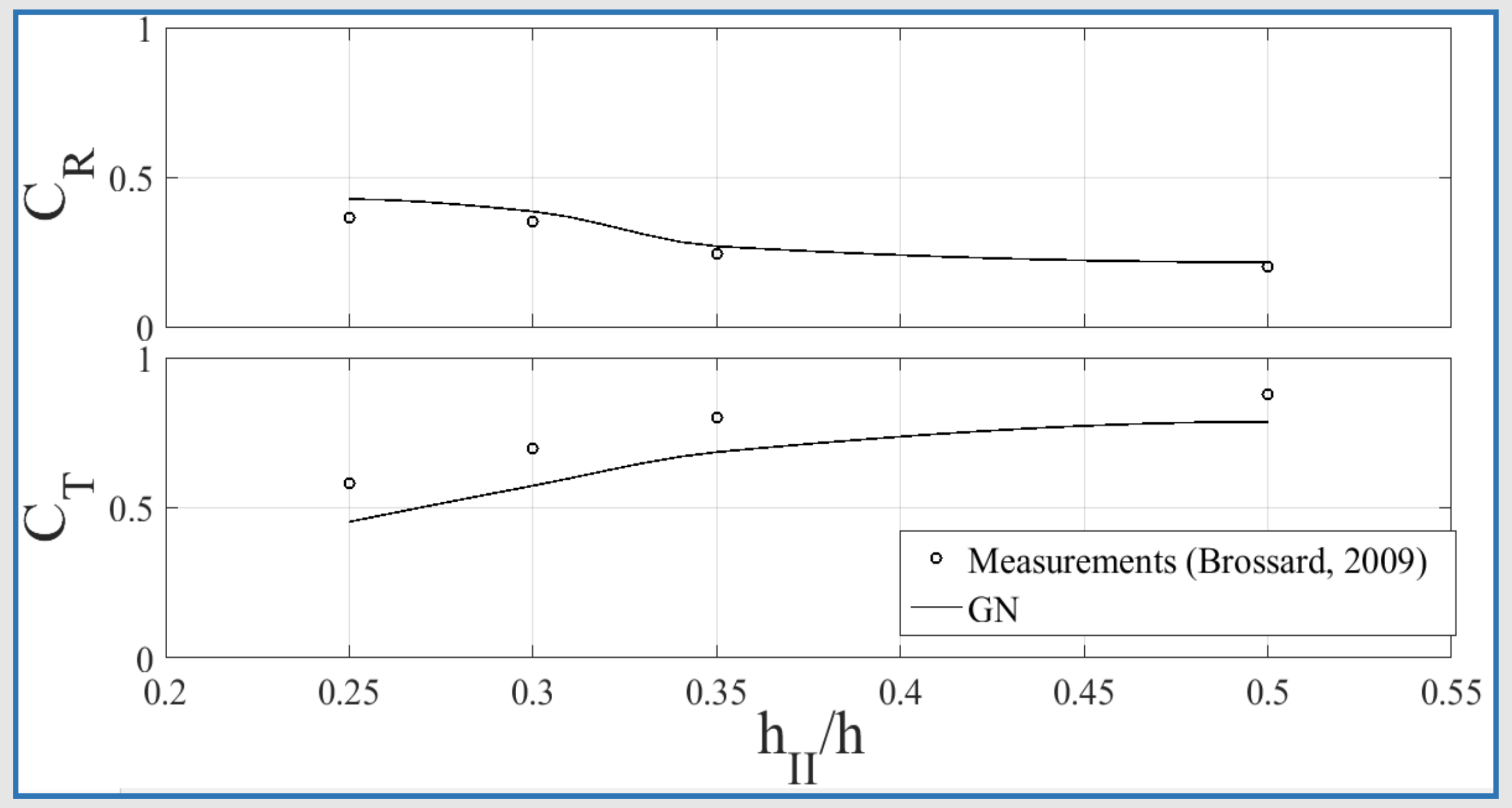

Fig 5: GN comparison of reflection coefficient, C_R, and transmission, coefficient C_T, as a function of h_{II}/h. Conditions are H/h = 0.1; h_{II}/h = Variable; B/h = 1.25; L/B = 2.5.

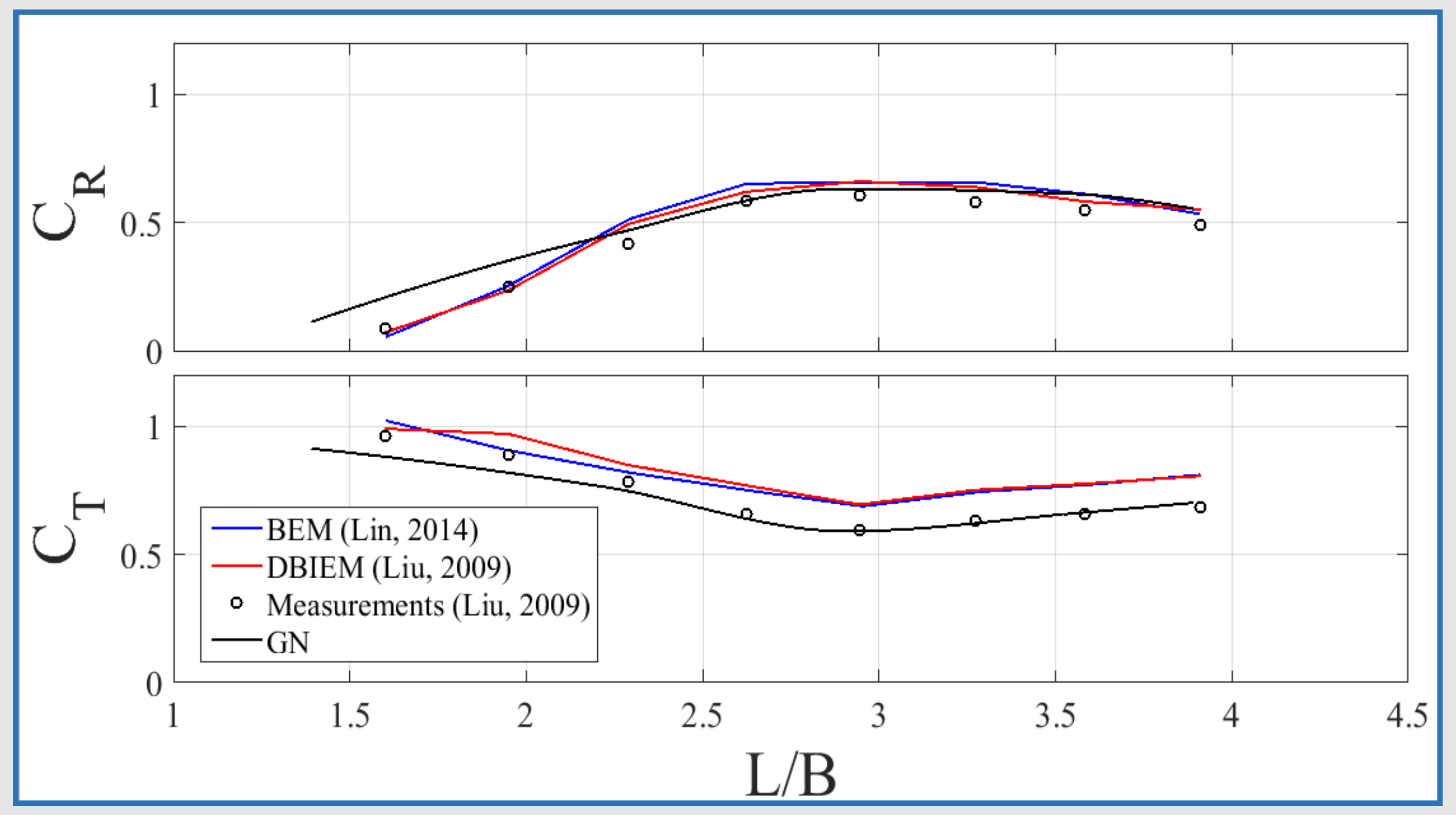

Fig. 6: GN comparison of reflection coefficient, C_R, and transmission, coefficient C_T, as a function of L/B. Conditions are H/h = 0.0667; h_{II}/h = 0.333; B/h = 2.0; L/B = Variable.

5. CASE STUDY: HURRICANE IKE (2008)

Here, we explore the possibility of wave scattering due to a submerged plate on incident shallow-water waves generated by Hurricane Ike (2008) off the Bolivar Peninsula coast, just north of Galveston Island. Figure 7 shows surface elevations of the waves, with and without the presence of a submerged plate.

The wave data is given by Blender et al. (2011) who obtained the data through a buoy at a water depth of h = 9.48m. The peak wave height, H = 5.49 m, and peak wave period, T = 11.75s. We choose h_{II}/h = 0.3 and L/B = 3 from the coefficient and transmission coefficient results to possibly produce significant reflection and small transmission without wave breaking. We compare the surface elevation of a simulated non-scattered transmitted wave when the submerged plate is absent, and the surface elevation of a simulated scattered transmitted wave with the presence of the plate during Hurricane Ike.

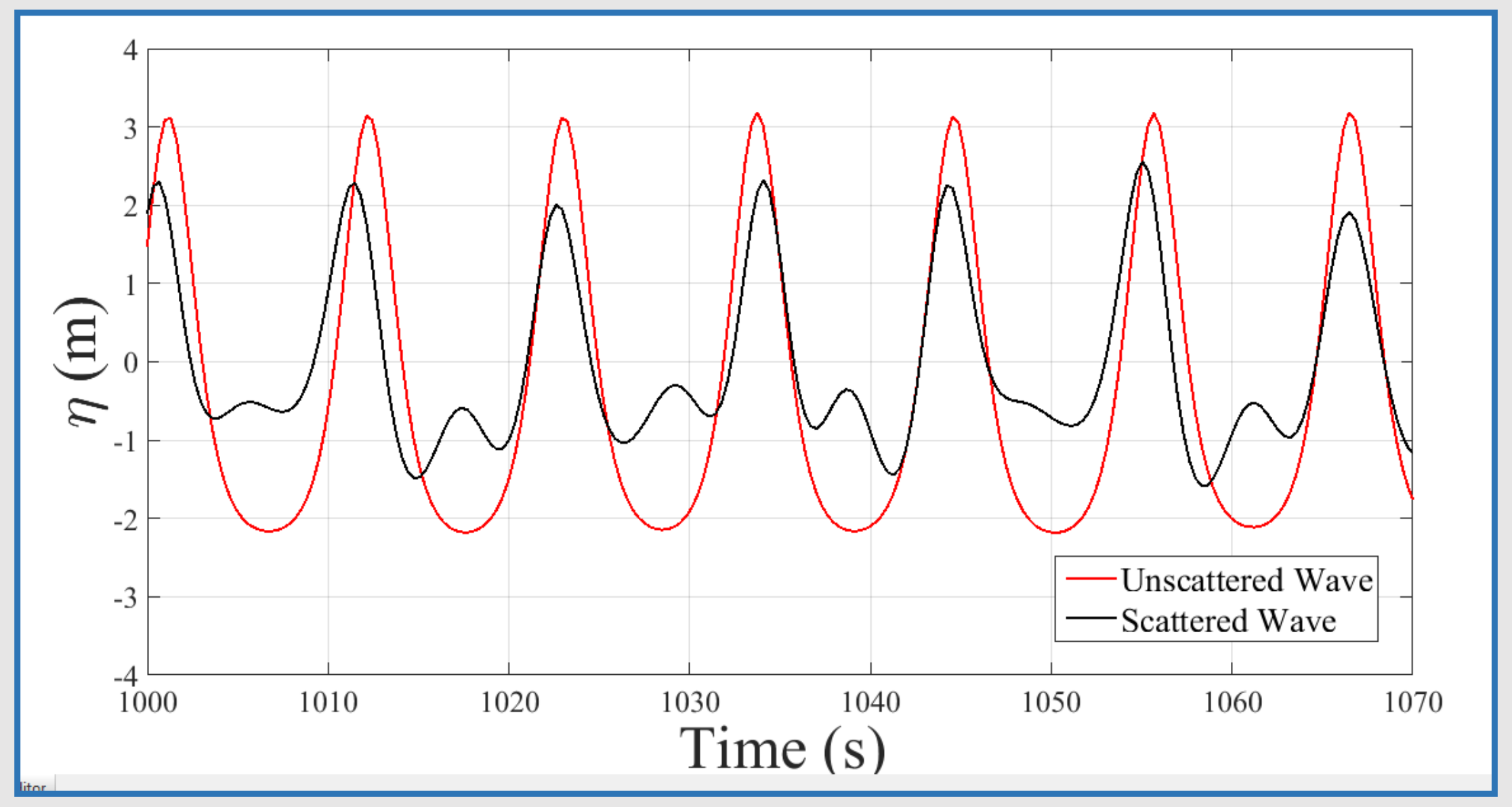

Fig 7: Case study of incident waves generated by Hurricane Ike (2008) over a submerged plate measured at 5B downwave. Conditions are: B/h = 3.74, H/h = 0.58, h_{II}/h = 0.25, L/B = 3, and L/h = 11.39.

The results show that the wave is significantly reduced, resulting wave height that is smaller by approximately 40 % - 45 %

6. CONCLUSION

Scattering of waves by a submerged horizontal plate is studied by use of the GN equation. A submerged horizontal plate is shown to be very effective in reflecting a significant portion of wave energy, while allowing some to transmit. Close agreement can be observed between GN equations and existing measurements and data. GN equations can be used for nonlinear wave scattering in shallow water conditions. C_R and C_T are shown to be significant due to a submerged plate, hence it can be used as a very simple and effective breakwater. A case study shows that the severity of damage due to waves generated by Hurricane Ike could be reduced by a submerged breakwater.

7. ACKNOWLEDGEMENTS

- Research Advisory Council Undergraduate Research Grant
- George Bush Presidential Library Foundation Undergraduate Travel Grant

8. REFERENCES

1. Bender, Christopher J., Jane M. Smith, and Andrew Kennedy. "Hurricane Ike (2008) nearshore waves: Simulations and measurements." *Coastal Engineering Proceedings* 1.32 (2011): 31.
2. Brossard, J., et al. "Higher harmonics induced by a submerged horizontal plate and a submerged rectangular step in a wave flume." *Coastal engineering* 56.1 (2009): 11-22.
3. Brossard, J., and M. Chagdali. "Experimental investigation of the harmonic generation by waves over a submerged plate." *Coastal Engineering* 42.4 (2001): 277-290.
4. Ertekin, Rifat Cengiz. *Soliton generation by moving disturbances in shallow water: theory, computation and experiment*. University of California, Berkeley, 1984.
5. Green, A. E., N. Laws, and P. M. Naghdi. "On the theory of water waves. "*Proceedings of the Royal Society of London A: Mathematical, Physical and Engineering Sciences*. Vol. 338. No. 1612. The Royal Society, 1974.
6. Green, Albert E., and Paul M. Naghdi. "A derivation of equations for wave propagation in water of variable depth." *Journal of Fluid Mechanics* 78.02 (1976a): 237-246.
7. Green, A. E., and P. M. Naghdi. "Directed fluid sheets." *Proceedings of the Royal Society of London A: Mathematical, Physical and Engineering Sciences*. Vol. 347. No. 1651. The Royal Society, 1976b
8. Grue, John. "Nonlinear water waves at a submerged obstacle or bottom topography." *Journal of Fluid Mechanics* 244 (1992): 455-476.
9. Hayatdavoodi, Masoud, Betsy Seiffert, and R. Cengiz Ertekin. "Experiments and calculations of cnoidal wave loads on a flat plate in shallow-water."*Journal of Ocean Engineering and Marine Energy* 1.1 (2015): 77-99.
10. Hayatdavoodi, Masoud, and R. Cengiz Ertekin. "Wave forces on a submerged horizontal plate-Part I: Theory and modelling." *Journal of Fluids and Structures* 54 (2015): 566-579.
11. Lin, Hong-Xing, et al. "Current effects on nonlinear wave scattering by a submerged plate." *Journal of Waterway, Port, Coastal, and Ocean Engineering* 140.5 (2014): 04014016.
12. Liu, Chunrong, Zhenhua Huang, and Soon Keat Tan. "Nonlinear scattering of non-breaking waves by a submerged horizontal plate: Experiments and simulations." *Ocean Engineering* 36.17 (2009): 1332-1345.