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HYDRODYNAMIC ANALYSIS OF A SUBMERGED WAVE ENERGY CONVERTER

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ABSTRACT

Hydrodynamic analysis of oscillation of a submerged plate due to nonlinear shallow water waves is studied. The submerged oscillating plate is the core component of a submerged wave energy converter. A concept design of the wave energy converter located in shallow water is presented. The device utilizes the heave motion of the submerged plate for wave energy extraction. The wave-induced loads on the oscillatory plate are obtained by use of the Level I Green-Naghdi nonlinear water wave equations, and by solving the Navier-Stokes equations using an open source CFD package. The nonlinear plate oscillations are obtained by solving the equation of vertical motion based on the space- and time-varying wave-induced forces. External spring and damping forces are applied to the plate as a means of controlling the plate oscillation and as a representation of the forces due to a power take-off converter. Plate response to the propagation of cnoidal waves are presented, including a study on the effects of plate oscillation with various damping and spring forces. Preliminary analysis of power output of the device is provided.

1 INTRODUCTION

Wave energy has the potential to alleviate the world's reliance on depleting fossil energy resources. The majority of wave energy converter (WEC) devices are comprised of at least one component that oscillates as a result of wave-induced forces. This oscillating component drives a piston or generator that converts the mechanical wave energy into electricity. Many WECs are located at or near the ocean surface, which are accompanied by major challenges. The extreme environmental loads inherent with large storms can damage WECs and necessitate regularly performed device maintenance, see e.g. [1]. In addition, large device fields located near shore result in unsightliness and interfere with recreational and shipping activities that occur on the surface in coastal regions, making it difficult to obtain necessary permits for installation and operation of the devices. Here, a completely submerged device is proposed to minimize the challenges that arise from surface-operating WECs. Some major challenges introduced by such a device include (but are not limited to) extracting comparable amounts of energy to surface-dwelling WECs, increased difficulty of installation and maintenance, and possible adverse effects to the natural sediment transport in and out of the region.

The oscillating component of the submerged device discussed in this paper consists of a thin, horizontal plate that can oscillate in only the vertical direction as a result of nonlinear, shallow water wave forces. Here, the Level I Green-Naghdi (GN) equations are utilized to calculate the wave-induced forces on the plate and to examine the wave interaction with the oscillating plate. In order to determine the plate oscillation, the GN equations are coupled with the equation of vertical motion of the plate and are solved simultaneously using numerical methods. Currently, a method for solving the plate oscillation using computational fluid dynamics (CFD) is being developed. Some discussion on this methodology is given in Section 3.2, however, in this paper, CFD is only used to verify the wave-induced forces given by the

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GN equations on a stationary plate. The CFD computations are performed by an open source program, namely OpenFOAM, which solves the Navier-Stokes equations.

The primary focus of this paper is on the hydrodynamic analysis of the oscillation of the submerged plate in shallow to intermediate water by use of the GN equations and the preliminary concept design of the device. In Section 2, the concept design of the submerged wave energy converter is presented. This is followed by discussion on the hydrodynamic analysis of motion of the submerged plate due to propagation of cnoidal waves in Section 3. Results and discussion on plate oscillation and power output are presented in Section 4. The paper is closed with some concluding remarks on the plate oscillation and the wave energy converter.

2 THE SUBMERGED WAVE ENERGY CONVERTER

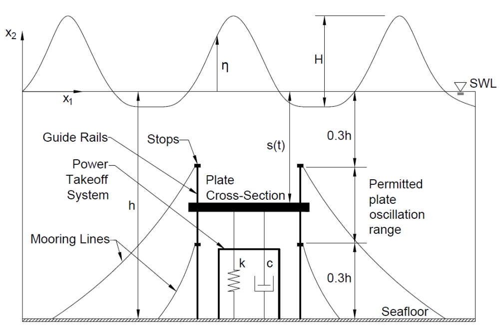

FIGURE 1. Schematic of the submerged WEC.

The submerged wave energy converter device consists of a flat, horizontal plate that is directly connected to a power-takeoff system located on the seafloor. The entire device is fully submerged at all times. A simplified schematic view of the device is shown in Fig. 1. The plate can only oscillate in the vertical direction. Thin rails are used to guide the vertical oscillation and restrict motion in all other directions. Frictional effects become apparent in this design component and are included in the hydrodynamic analysis of the plate motion.

Physical stops are incorporated into the design in order to limit the plate oscillation to a desired range. The power-takeoff system is located under the plate on the seafloor and will follow the same oscillation as the plate (direct-drive system). Hence, a limit is set for the oscillation range of the plate. In this study, the stops are located so that $s(t)$, the submergence depth shown in Fig. 1, is shallowest at 0.3 of the water depth and deepest at 0.7 of the water depth from the still water level (SWL). The stop locations can be modified and optimized for the specific power-takeoff system that is to be used. Specific requirements on the size of the guide rails and power-takeoff system may result in these components being large enough to significatively alter the flow field around the plate. In this work, it is assumed that effect of the guide rails and the power-takeoff system are on the flow field is negligible.

In this preliminary design, the impact of the power-takeoff system on the plate is simplified to a spring and damper system. In order to optimize the design, various coefficient values will be studied to yield an oscillation that results in maximized power output. Desirable spring and damping coefficients are those that cause the oscillation to achieve its maximum displacement just before it reaches the stop limits. This will ensure that the plate follows the wave frequency closely and does not remain at the stop for long periods of time, which would reduce the amount of energy the device can harvest. The plate can be buoyant, although this is not a general requirement.

3 HYDRODYNAMIC ANALYSIS

The submerged WEC is composed of a thin plate that oscillates vertically due to the wave-induced forces, and restricted from moving in any other direction. The plate is rigid, and is to remain horizontal and completely submerged throughout its oscillation. Further complexities arise with an oscillating plate, as apposed to a fixed one, since the wave forces and the diffracted surface elevation will vary with the instantaneous submergence depth. In this study, we will solve the problem of wave-induced plate motion by use of the Level I GN equations and verify the wave-induced forces by applying CFD to solve the Navier-Stokes equations.

3.1 The Green-Naghdi Equations

[2] developed a theory of nonlinear wave propagation using directed fluid sheets known as the Green-Naghdi equations. A two-dimensional rectangular Cartesian coordinate system is used where the origin is located on the SWL with x_1 directed to the right and x_2 directed upward. The water depth is h, the surface elevation is $\eta(x_1,t)$, and the wave height is H. Assumptions commonly made in other wave theories, such as an inviscid fluid or irrotational flow, are not made in the general form of this theory. Free-surface and seafloor boundary conditions are satisfied exactly and the integrated form of mass, momentum, and energy are conserved. The GN equations are classified by *levels*, which correspond to the variation of vertical velocity in the x_2 direction. In this study, we consider the Level I GN equations, in which fluid vertical velocity varies

linearly along the water column, i.e. $\frac{\partial u_2}{\partial x_2} = C(x_1,t)$. By applying mass and momentum conservation laws, the final form of the Level I GN equations for the two-dimensional propagation of an incompressible and inviscid fluid over a time-invariant seafloor are given as

$$\eta_{,t} + [(h+\eta-\alpha)u_1]_{,x_1} = 0, \tag{1}$$

$$\dot{u}_1 + g\eta_{,x_1} + \frac{\hat{p}_{,x_1}}{\rho} = -\frac{1}{6}\big\{[2\eta+\alpha]_{,x_1}\ddot{\alpha} + [4\eta-\alpha]_{,x_1}\ddot{\eta} + (h+\eta-\alpha)[\ddot{\alpha}+2\ddot{\eta}]_{,x_1}\big\}, \tag{2}$$

where u_1 is the horizontal velocity, $\hat{p}(x_1,t)$ is the pressure at the top surface, $\alpha(x_1)$ is the curve bounding the bottom of the fluid sheet, ρ is the fluid density and g is the gravitational acceleration [3]. The dot and double dot notation in Eq. (1b) represents the two-dimensional first and second substantial derivative, respectively. In this study, we are only concerned with the Level I GN equations; see [3] and [4] for further developments of the GN equations. For a more complete derivation of the general GN equations, see [5].

Recently, [6] obtained the forces on a fixed, submerged, horizontal plate using the GN equations. A similar method is followed here to determine the wave-induced vertical (F_{x2}) and horizontal (F_{x1}) forces on the submerged oscillating plate, see [7] for details.

Position of the plate at each time step is found by coupling the equation of motion of the plate in the vertical direction, with the fluid governing equations, Eqs. 1 and 2. The equation of vertical motion of the plate is given by Newton's second law:

$$\Sigma F = m s_{,tt}\,, \tag{3}$$

where m is the mass per unit length of the two-dimensional plate, $s_{,tt}$ is the instantaneous plate acceleration, and ΣF is the summation of all vertically acting forces on the plate. For this study, the force summation consists of the following:

$$\Sigma F = F_{x2} + F_f + F_s + F_d + F_{st}, \tag{4}$$

where F_{x2} is the vertical wave force, F_f is the frictional force, F_s is the spring force, F_d is the damping force, and F_{st} is the difference between the buoyant and weight force of the plate. The frictional force due to motion of the plate on the guide rails is given by:

$$F_f(x_2,t) = -\left(\frac{|s_{,t}|}{s_{,t}}\right)\mu F_{x1}, \tag{5}$$

where $s_{,t}$ is the instantaneous plate velocity, μ is the constant coefficient of friction, and $F_{x1}(x_2,t)$ is the instantaneous horizontal wave force on the plate. The fraction appearing first in the equation is used to ensure that the frictional force always acts opposite to the vertical plate velocity. The spring and damping forces are assumed to vary linearly with displacement and plate velocity, respectively. The spring force is given by:

$$F_s(x_2,t) = -k(|s_0| - |s(t)|), \tag{6}$$

where k is the spring stiffness, s_0 is the initial plate submergence depth. The damping force is given by:

$$F_d(x_2,t) = -c s_{,t}\,, \tag{7}$$

where c is the damping coefficient, and $s_{,t}$ is the vertical plate velocity.

The equations of fluid motion, Eqs. 1 and 2, which results in the wave-induced forces, and the equation of vertical plate motion are coupled and are solved simultaneously since F_{x2} and F_{x1} vary with both time and position. The fluid system of equations is numerically solved by a second-order finite difference approach and by a modified Euler method for the time integration; see [8] for more information. The equation of motion of the plate is solved numerically, using the 4th-order Runge-Kutta method. Computational time for the coupled system is approximately one minute.

The mechanical power that is available to the generator is determined using the plate motion. The attachment between the plate and the oscillatory component of the power-takeoff system is rigid. Hence, the translator component of the generator follows the plate oscillation. The time rate of work done by the viscous damper is the only power that is available to the generator for energy conversion. The instantaneous power that is available to the power-takeoff system, before any mechanical energy is converted into electricity, is given by

$$P(x_2,t) = F_d s_{,t}\,. \tag{8}$$

The efficiency of the power take-off system itself must be determined to obtain the final power output of the device, however this is beyond the scope of this study.

3.2 Navier-Stokes Equations

The wave-iduced forces on a fixed plate are also determined by employing the Navier-Stokes equations via the open-source CFD package, OpenFOAM. A two-dimensional numerical wave tank is constructed and discretized with a non-uniform mesh, finer around the plate and the free surface. The domain is 18 meters long with a height of 1.6 meters and is discretized into 45,000 cells. A grid convergence test has shown that accurate results are attained with this grid density. No-flux boundary conditions are applied to the front, back, and bottom walls to ensure no fluid escapes the tank and an atmospheric pressure condition is enforced on the top face. Additionally, the plate is added to a particular depth below the free-surface and no-slip conditions are applied to each face. As there are two fluids present in the tank (water and air), a volume of fluid method is used to track the fluid interface.

A dynamic mesh is implemented to track the oscillation of the plate by means of a dynamic mesh solver, namely interDyMFoam. To achieve proper mesh morphing, the equations of motion for the plate are solved by calling the sixDoFRigidBodyMotion solver inside of interDyMFoam. This solver calculates the linear and angular motion of a solid body in six degrees of freedom based on prescribed plate properties such as mass, center of mass, and moment of inertia. In this study, the plate is assumed to be restricted to vertical motion by the device guide rails, therefore appropriate constraints are applied within the solver. The sixDoFRigidBodyMotion solver iterates within interDyMFoam to converge on a new plate position for each time-step. Computational time for this particular case is approximately six hours.

4 RESULTS and DISCUSSION

The cnoidal wave forces on a submerged plate given by the GN equations are compared with CFD and laboratory measurements from [9] in Fig. 2. In this case, the submerged plate is fixed so that the accuracy of the GN solution can be assessed. Overall, a close agreement is observed between the GN forces and those found by CFD and laboratory measurements.

Next, the results of the plate oscillation due to incident cnoidal waves are presented. The following results were obtained by the GN solution discussed in Section 3.1. Here, we assume a plate width $\bar{B} = B/h = 0.2$, initial submergence depth $\bar{s}_0 = s_0/h =$ -0.5, plate thickness $\bar{t}_p = t_p/h = 0.025$, and plate mass $\bar{m} = m/(\rho h^2 L_p) = 0.05$, where L_p is the plate length into the page. The dimensionless spring and damping coefficients are $\bar{k} = k/(\rho g h L_p) = 0.08$ and $\bar{c} = c/(\rho\sqrt{g}h^{1.5}L_p) = 0.03$. The dimensionless wave parameters are wavelength $\bar{\lambda} = \lambda/h = 10$ and wave height $\bar{H} = H/h = 0.2$. Time is nondimensionalized as $\bar{t} = t\sqrt{g/h}$.

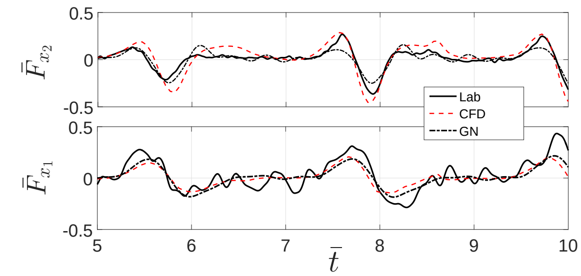

FIGURE 2. Comparison of cnoidal wave horizontal ($\bar{F}_{x_1} = F_{x_1}/\rho g h t_p L_p$) and vertical ($\bar{F}_{x_2} = F_{x_2}/\rho g h^2 L_p$) wave forces on a submerged plate, calculated by use of OpenFOAM and the GN equations vs. laboratory measurement of [9] ($h = 0.071$m, $H/h \approx 0.3$, $\lambda = 1.9$m and $s/h = 0.6$). $L_P = 14.9$ cm and $t_P = 1.27$ cm are plate length (into the page) and thickness, respectively and $B = 30.5$ cm is the plate width, parallel to wave propagation direction.

The plate oscillation, $\bar{s}(t)$, for various spring stiffness, $\bar{k}$, and damping coefficient, $\bar{c}$, values are shown in Figs. 3 and 4, respectively. In Fig. 3, the aforementioned parameters remain the same except for a variation in $\bar{k}$. It is shown that the amplitude of plate oscillation increases with decreasing $\bar{k}$. Fig. 4 is created in a similar manner; however $\bar{c}$ is the only varying parameter. Comparing Figs. 3 and 4, we notice that varying $\bar{k}$ has a more profound effect on the oscillation amplitude than varying $\bar{c}$, the damping effect due to the power-takeoff system.

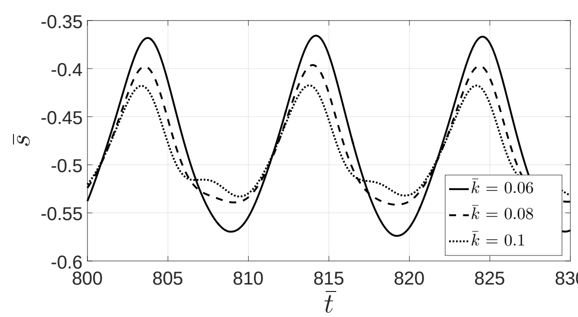

FIGURE 3. Time history of plate oscillation for various $\bar{k}$.

The root mean square (RMS) power that is provided to the generator over a period of $\bar{t} = 800$ to $\bar{t} = 1000$ for various $\bar{H}$ and $\bar{\lambda}$ values is shown in Table 1. All other values are the same as the aforementioned ones. Here, we dimensionalize power to give units of Watts per unit meter of plate length using $P = \bar{P}\rho g h^2\sqrt{gh}$, where $\bar{P}$ is dimensionless power. Di-

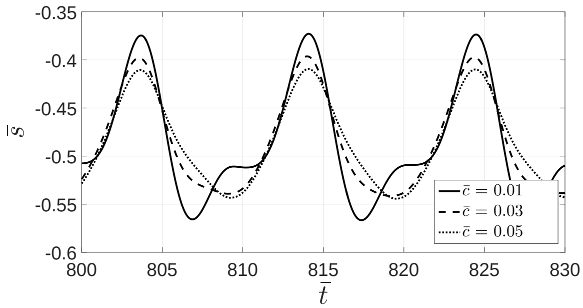

FIGURE 4. Time history of plate oscillation for various $\bar{c}$.

TABLE 1. RMS power matrix in kilo Watts per unit length of the plate for various $\bar{H}$ and $\bar{\lambda}$ values.

		$\bar{H}$			
		0.1	**0.2**	**0.3**	**0.4**
$\bar{\lambda}$	**15**	0.415	1.17	1.73	2.04
	20	0.963	3.69	5.46	5.67
	25	0.225	1.08	2.04	1.98
	30	0.0635	0.447	1.26	2.09

mensional values used in this case are $\rho = 1000kg/m^3$, $g = 9.81m/s^2$, and $h = 10m$. It can be seen that power generally increases with increasing $\bar{H}$ and increases from $\bar{\lambda} = 5$ to $\bar{\lambda} = 9$, but then mostly decreases with increasing $\bar{\lambda}$ thereafter.

5 CONCLUSIONS

The hydrodynamic analysis and preliminary design of a fully submerged wave energy converter located in shallow to intermediate water depth is discussed. It is shown that the motion of the submerged plate can be determined by solving the GN nonlinear shallow water wave equations and, eventually, by use of CFD. Computations of the nonlinear, shallow water GN equations, solved by use of a finite-difference approach, take approximately one minute on a typical single CPU computer. This allows for assessment of variation of power output with many combinations of wave-conditions and device geometries, i.e., the GN model can be used to obtain an optimized device for given conditions. The CFD computations, on the other hand, take several hours to several days on a typical workstation, depending on the incoming wave length and plate geometry. The CFD computations will allow for a better understanding of the flow field around the plate, including formations of vortices and possible system vibrations.

A direct-drive power take-off system, modeled by a spring-damper system, is used to convert the oscillating plate's mechanical energy into electrical energy. The external spring and damping effects can be modified to optimize the plate oscillation for maximum power output under design conditions. The device is very simple, is fully submerged and independent of direction of incoming waves. An array of devices can be installed with little to no reduction on power output of individual units.

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