Green Water Effect on Ship Motions and Loads via Dam Breaking Model

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Highlights
An efficient, low-fidelity methodology is presented that applies classical shallow water theory and a dam breaking formulation to approximate the green water on deck problem.

1 Introduction

The green water (GW) effect refers to seawater flowing onto a ship's deck as the incident wave height exceeds the deck level. Depending on the hull form, ship speed and seaway conditions, the GW could happen in different shapes and forms with distinctive characteristics such as breaking waves, water sloshing and impact, hydraulic jump, 3-dimensional flows and dynamic inflow-outflow conditions. The modeling of the GW effect can have a significant impact on the predicted motions of surface ships. This is particularly true for vessels with a large foredeck area, where the static and dynamic loading of GW can affect statistics of extreme motions and the occurrence of stability failures. To model the GW effect more accurately, high fidelity computational fluid dynamics (CFD) tools are often required. However, in practice, especially during the early-stage design phase, tens of thousands of numerical simulations need to be performed in a short time for various designs and operation conditions. Therefore, development of an efficient approach to account for the GW effect without sacrificing too much physics and accuracy is necessary.

The objective of this study is to develop a strip theory-like approach for predicting the occurrence of GW on deck, which should be both sufficiently fast and accurate to be incorporated into potential flow codes or reduced order codes for ship motion and load simulations. More specifically, this study incorporates the classical one-dimensional (1-D, one independent spatial dimension) dam breaking model [1] into an existing potential-flow based ship hydrodynamics code. Although the 1-D modeling is inherently less accurate than the 3-D CFD calculations [2], it is physics-based and much more computationally efficient. It reflects the fundamental physics in the dam breaking problem based on the nonlinear shallow water theory and has an analytical solution in closed form that allows for much faster execution of the GW on deck calculations. The solution should have sufficient accuracy and acceptable agreement with higher fidelity data (model tests, CFD, etc.) in simplified GW on deck scenarios.

2 Approach

The motion of green water on ship deck can be characterized by shallow water theory and described by a dam breaking formulation. Since the ship deck has 6-degrees-of-freedom (6-DOF) and the green water flow relative to the ship deck is of major concern, the classical dam breaking formulation derived with respect to the earth-fixed coordinated system [1] needs to be transformed with respect to the ship-fixed (or deck-fixed) coordinate system. Figure 1 illustrates the two coordinate systems, the earth-fixed *O-XYZ* and the deck-fixed *o-xyz* with the free surface $\zeta(x, y, t)$ and the deck surface $b(x, y)$ expressed in the *o-xyz* frame. The instantaneous position and orientation of o-xyz relative to *O-XYZ* is prescribed by the position of *o-xyz* origin and its Euler angles (ϕ, θ, ψ) for roll, pitch and yaw, respectively.

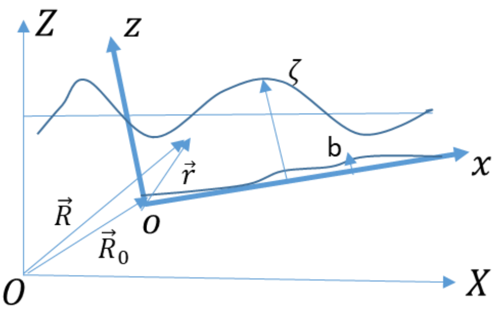


Figure 1. Free surface and the bottom surface profile expressed in o-xyz frame fixed on ship deck.

In the *o-xyz* frame, the momentum equation in vector form is written as:

$$\frac{\partial}{\partial t}\vec{v} + (\vec{v}\cdot\nabla)\vec{v} = \vec{f} - \vec{a} - \frac{1}{\rho}\nabla p \qquad (1)$$

where $\vec{v} = (u, v, w)$ is the relative fluid velocity vector expressed in component form in the *o-xyz* frame and the body force $\vec{f} = \vec{g}$ is gravity, $\vec{a} = \ddot{\vec{R}}_o + \dot{\vec{\omega}} \times \vec{r} + \vec{\omega} \times (\vec{\omega} \times \vec{r}) + 2\vec{\omega} \times \vec{v}$ is the acceleration associated with the rigid body motion of the *o-xyz* frame. The variables ρ and p correspond to the fluid density and pressure, respectively. The continuity equation and the boundary conditions on the free surface and on the bottom surface remain the same in the *o-xyz* frame as in the *O-XYZ* frame. Adopting the shallow water assumption, that the typical length scale d in the vertical direction is samller than the typical length scale l in the horizonal direction, i.e., $\varepsilon = \frac{d}{l} \ll 1,$ and including the contribution of $\vec{a}$ to the forcing term and the rotation of the *o-xyz* frame, the 2-D (two independent spatial dimensions) shallow water equations and the the lowest order free surface boundary condition can be derived through a perturbation expansion, which can be further reduced to the 1-D shallow water system in two equivalent forms, as follows:

$$\begin{cases} u_t + uu_x + (a_k + gcos\theta cos\phi)\zeta_x = 0 \\ \zeta_t + [u(\zeta - b)]_x = 0 \qquad at\ z = \zeta \end{cases} \leftrightarrow \begin{cases} u_t + uu_x + 2cc_x + gb_x = 0 \\ 2c_t + 2uc_x + cu_x = 0 \end{cases} \qquad (2)$$

where $c = \sqrt{(a_k + gcos\theta cos\phi)(\zeta - b)}$ is the disturbance wave speed, $a_k = \ddot{Z}_0 cos\theta - \ddot{X}_0 sin\theta$ is the acceleration of the *o-xyz* frame in the z-direction. The c reduces to the conventional form $c = \sqrt{g(\zeta - b)}$ if *o-xyz* is fixed with respect to *O-XYZ*. The nonlinear system of Equation (2) can be solved by the characteristics method [1] for $u(x, t)$ and $\zeta(x, t)$ from which the the lowest order pressure on the deck can be calculated as the hydrostatic pressure $p = (a_k + gcos\theta cos\phi)[\zeta(x, y, t) - b]$. The derivation above suggests that replacing g in the wave speed c with $G = \ddot{Z}_0 cos\theta - \ddot{X}_0 sin\theta + gcos\theta cos\phi$, the classical 1-D dam breaking formulation derived in *O-XYZ* can be used in *o-xyz* frame.

3 Formulation

Two dam breaking formulations exist, based on the initial downstream water depth prior to the dam break, each of which reflects distinctive physical phenomena. When the downsteam water depth $h_0 = 0,$ the formulation for so-called "dry" dam breaking involves depresion disturbance waves only, and the resulting fluid flow is continuous. However, when $h_0 > 0$, the fluid flow downstream is discontinuous, and the formulation for "wet" dam breaking accounts for both the depression and the shock waves (hydraulic jump/bore). The wet dam breaking case is illustrated in Figure 2. The dam is initially located at $x = 0$, $t = 0$, and breaks at $t = 0+$

towards the downstream side, $x > 0$. The water depths upstream and downstream of the dam are h_1 and h_0, respectively, and assumed to be constants. After the dam breaks, for $t > 0$, the fluid domain is characteristically divided into four zones that involve discontinuities of the flow and the shock wave. This includes the undisturbed downstream zone (0), the constant flow zone (2) with the downstream propagating shock wave, the depression wave zone (3) and the undisturbed upstream zone (1).

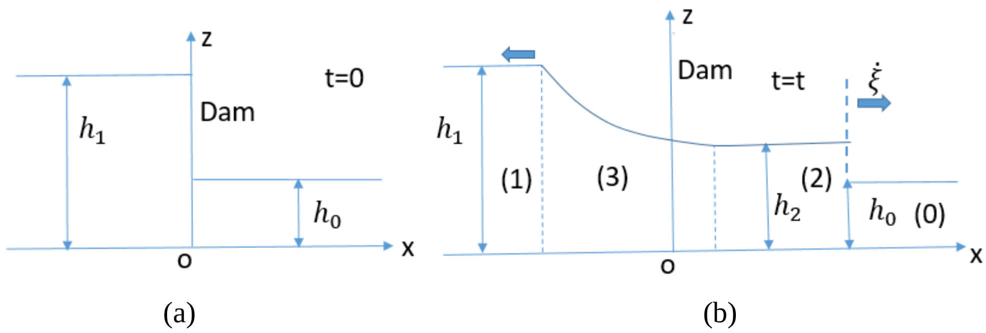


Figure 2. 1-D dam breaking diagram with non-zero initial water height downstream, (a) before breaking, (b) after breaking.

The formulation given in the earth-fixed frame [1] is extended to the ship-fixed frame, so that the effect of the ship motion such as vertical acceleration, rotation in pitch and roll is included in the GW calculations. Given the water heights on two sides of the dam before breaking, h_1 and h_0, first the analytical solutions are obtained in zone (2) and (3), where the shock speed $\dot{\xi}$ can be obtained by solving the transcendental equation:

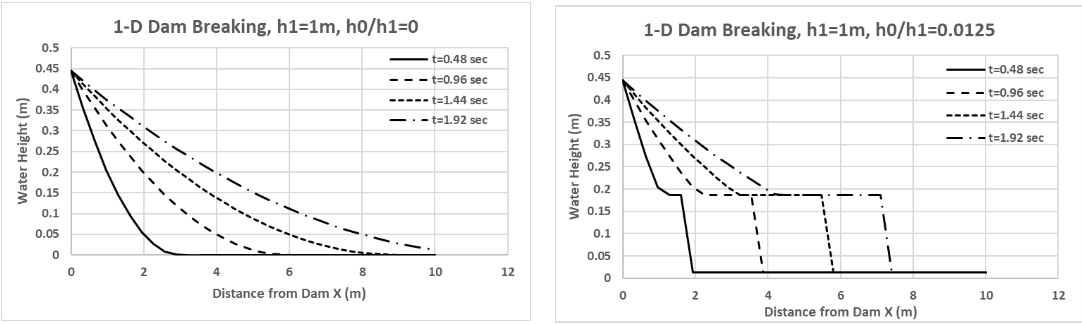
$$\frac{\dot{\xi}^2}{C_0^2} - \frac{\left(1+\sqrt{1+8(\dot{\xi}/C_0)^2}\right)}{4} + 2\left(\sqrt{\frac{-1+\sqrt{1+8(\dot{\xi}/C_0)^2}}{2}} - \frac{C_1}{C_0}\right)\frac{\dot{\xi}}{C_0} = 0 \tag{3}$$

where the speed of the shallow water wave is $C_i = \sqrt{Gh_i},\ i = 0, 1$. The formulation and solutions derived above is verified first against the existing solutions [1], and then they are implemented as one of the GW calculation options in the Large Amplitude Ship Motions (LAMP) program [3]. For the head sea case, a series of strips parallel to each other and to the ship centerline is defined on the ship's deck, and the GW flow is assumed to be flowing from the forward deck edge aft. This head sea case scenario is given a top priority, since it represents the most common and severe GW effect on ship motions and sectional loads. For oblique seas, the strips may be oriented along the primary wave direction.

4 Results

With the solution of $\dot{\xi}/c_0$ from Equation (3), the speeds of the shallow water waves and fluid in zone (2) and zone (3) can be obtained. Figure 3 shows the water height at four time instants for initial water height ratio h_0/h_1= 0 and h_0/h_1= 0.0125, respectively. Figure 3(a) corresponds to the dry deck case with the downstream water height $h_0 = 0$ and the fluid flow is continuous with the depression wave propagating up and downstream. Figure 3(b) is the wet deck case with the initial water height downstream $h_0 > 0$ and the fluid flow is discontinuous with the shock wave propagating faster than both the fluid speed u_2 and the wave speed c_2 in zone (2). The results reveal that the speed of the wave front travels faster in the dry deck case than in the wet deck case even though the ratio h_0/h_1 = 0.0125 is very small. In most of the real world

GW events, a shock wave is involved; therefore, assuming a certain amount of the downstream water depth pre-dam breaking may be more realistic.



(a) (b)

Figure 3. Water height at different times for (a) $h_0/h_1 = 0$ and (b) h_0/h_1=0.0125 with h_1=1 m.

The 1-D dam breaking model shown above has been implemented in LAMP and applied to simulate the GW on deck for David Taylor Model Basin (DTMB) Model 5415 running at 10 knots in a Sea State 8 (H_s = 14 m, T_p = 14.2 s) Bretschneider spectrum, with long crested, head seas. The GW calculation starts along each strip when the vertical relative motion between the wave surface and the forward deck edge exceeds a threshold value. A snapshot of the free surface profile at a time instant from a preliminary simulation is presented in Figure 4, where the incident wave surface is in transparent light blue and the GW height on deck, which is flowing aft, is in cyan.

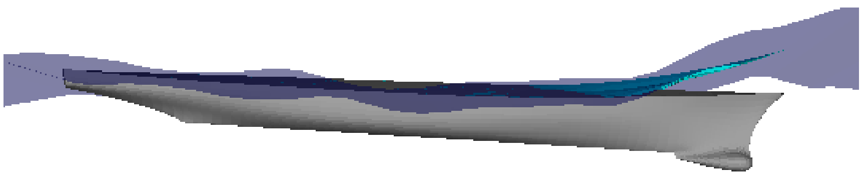


Figure 4. The GW profile at time = 206.70 s for DTMB model 5415 running at 10 knots in Sea State 8, long crested, head seas.

Further research will include (a) the inflow (upsteam) and outflow (downstream) boundary condition of the GW flowing on and off deck edge, (b) GW impacting on a solid wall on deck, (c) oblique sea conditions and (d) evaluation of the pros and cons of the 1-D dam breaking model as compared with high fidelity methods.

5 Acknowledgements

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