Electromagnetic Precursors of Tsunamis

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1 Introduction

Recent observations by submarine geomagnetic observatories revealed the existence of a characteristic electromagnetic signature anticipating the arrival of earthquake-generated tsunamis in several locations around the world (Lin et al. 2021). This captivating phenomenon arises from the dynamo effect, where the motion of conductive seawater through Earth's primary magnetic field gives rise to a small electromagnetic (EM) field.

This paper presents an analytical model combining potential flow and dynamo theories to explain the observed phenomenon. We introduce a rigorous mathematical framework via the governing Cauchy–Poisson boundary-value problem associated with surface gravity waves and the EM field arising from a disturbance in the seabed. Through the application of asymptotic analysis, we demonstrate that the EM signal, observed at significant distances from the epicenter, can be decomposed into two components. The first term is proportional to the Airy function, propagating concurrently with the surface gravity wave. The second term is proportional to the Scorer function, displaying a phase lag relative to the surface gravity wave. This phase lag provides an explanation for the observed time discrepancy between the arrival of the EM signal and the surface gravity wave resulting from seabed deformation, as evidenced in field measurements and numerical findings (Minami et al. 2015).

Further analysis investigating the parametric behaviour of the system will be presented at the Workshop.

2 Mathematical Model

Consider an infinite ocean with a horizontal seabed. Establish a Cartesian coordinate system, aligning the x-axis horizontally and the z-axis vertically from the undisturbed free surface. The seabed is at $z = -h$. The y-axis is perpendicular to the (x, z) plane and t represents time.

Consider a linearised potential flow model for the ocean, where the velocity potential satisfies the Laplace equation

$$\nabla^2\Phi = 0, \quad z \in (-h, 0), \tag{1}$$

the kinematic boundary condition on the free surface

$$\frac{\partial \Phi}{\partial z} = \frac{\partial \zeta}{\partial t}, \quad z = 0, \tag{2}$$

the dynamic boundary condition on the free surface

$$\frac{\partial \Phi}{\partial t} + g\zeta = 0, \quad z = 0, \tag{3}$$

and the dynamic condition on the seabed

$$\frac{\partial \Phi}{\partial z} = W(x, y, t), \quad z = -h, \tag{4}$$

where $\zeta(x, y, t)$ is the free-surface elevation, g is gravity, and $W(x, y, t)$ is the vertical speed of motion of the seabed displacement modelling an earthquake. Let Φ and $\nabla\Phi$ decay as $(x, y) \to \pm\infty$. The seabed motion starts at $t = 0^+$, hence we request that the system be at rest for $t \leq 0^-$.

The EM field induced by the tsunami is governed by the dynamo equation

$$\frac{\partial \mathbf{b}}{\partial t} - \eta \nabla^2 \mathbf{b} = \mathbf{F} \cdot \nabla \mathbf{u}, \tag{5}$$

where $\mathbf{b}$ is the magnetic flux density (T), η is the constant magnetic diffusivity ($\mathrm{m}^2\mathrm{s}^{-1}$), $\mathbf{F}$ is the steady Earth's field and $\mathbf{u} = \nabla\Phi$ is the velocity field in the fluid.

Solution of the gravity wave (tsunami) for a sudden displacement with speed $W(x, t) = H_0(x)\delta(t)$ is straightforward, as presented in Mei et al. (2005). Using a combined Laplace-Fourier transform and integration in the complex plane yields the free-surface elevation

$$\zeta(x, t) = \frac{1}{4\pi} \int_{-\infty}^{+\infty} \frac{\tilde{H}_0(k)}{\cosh(kh)} \left[e^{i(kx - \omega t)} + e^{i(kx + \omega t)} \right] dk, \tag{6}$$

where $\omega^2 = gk \tanh(kh)$, g is gravity, and the tilde denotes the Fourier transform along x.

Employing a similar technique for the EM field, Renzi & Mazza (2023) showed that the vertical EM component b_z is made by two parts,

$$b_z(x, -h, t) = b_z^o(x, t) + b_z^e(x, t). \tag{7}$$

In the latter,

$$\begin{aligned} b_z^o(x, t) = \quad & -\frac{1}{4\pi} \int_{-\infty}^{+\infty} \frac{|k|\, \tilde{H}_0\, f_{z,-}^{(h)}\, e^{i(kx - \omega t)}}{\omega^2 \left\{ 2|k|\alpha_- \cosh(\alpha_- h) + (k^2 + \alpha_-^2) \sinh(\alpha_- h) \right\}} dk \\ & -\frac{1}{4\pi} \int_{-\infty}^{+\infty} \frac{|k|\, \tilde{H}_0\, f_{z,+}^{(h)}\, e^{i(kx + \omega t)}}{\omega^2 \left\{ 2|k|\alpha_+ \cosh(\alpha_+ h) + (k^2 + \alpha_+^2) \sinh(\alpha_+ h) \right\}} dk, \end{aligned} \tag{8}$$

is a transient oscillatory term, whereas

$$b_z^e(x, t) = \frac{1}{2\pi} \sum_{n=1}^{+\infty} \int_{-\infty}^{+\infty} \frac{|k|\, \tilde{H}_0\, f_{z,n}^{(h)}\, e^{ikx}}{d_n(k)} e^{-\eta\left(k^2 + \beta_n^2\right)t}\, dk, \tag{9}$$

is a fast-decaying evanescent term. Detailed expressions for the complex coefficients $f_{z,\pm}^{(h)}(k)$, $f_{z,n}^{(h)}(k)$, $\alpha_\pm(k)$, and $\beta_n(k)$ are presented in Renzi & Mazza (2023).

3 Asymptotic solution

We consider an asymptotic solution for the oscillatory component of the vertical EM field at large time after the earthquake. For the sake of example, consider an observer at some point $x > 0$. Away from the epicenter, only right-going waves survive. Expanding the argument of (8) up to $O(k^3)$ and using the method of stationary phase yields a novel asymptotic formula for b_z^o, namely

$$b_z^o(x,t) = m_z^o(x,t) + g_z^o(x,t). \tag{10}$$

In the latter,

$$m_z^o(x,t) = \frac{F_z}{2} \frac{\tilde{H}_0(0)/h}{gh + (2\eta/h)^2} \left(\frac{2g}{ht}\right)^{1/3} \left\{ \frac{2\eta}{h} \, \mathrm{Gi} \left[\left(\frac{2}{\sqrt{gh} h^2 t} \right)^{1/3} \left(x - \sqrt{gh}\, t \right) \right] \right\}, \tag{11}$$

and

$$g_z^o(x,t) = \frac{F_z}{2} \frac{\tilde{H}_0(0)/h}{gh + (2\eta/h)^2} \left(\frac{2g}{ht}\right)^{1/3} \left\{ \sqrt{gh}\, \mathrm{Ai} \left[\left(\frac{2}{\sqrt{gh}\, h^2 t} \right)^{1/3} \left(x - \sqrt{gh}\, t \right) \right] \right\}. \tag{12}$$

In (11), Ai is the well-known Airy function, whereas Gi is the Scorer function

$$\mathrm{Gi}(Z) = \frac{1}{\pi} \int_0^{+\infty} \sin\left(\frac{t^3}{3} + Zt \right) dt. \tag{13}$$

Expressions (10)–(12) indicate that the oscillatory part of the magnetic field diminishes at a rate $O(t^{-1/3})$. Consequently, its decay rate matches that of the dominant gravity wave (Mei et al. 2005). Equations (10)–(12) further unveil that the magnitude of the asymptotic magnetic field is directly proportional to the seabed deformation area $\tilde{H}_0(0)$, and thus linked to the vertical displacement.

It is important to note that b_z^o comprises two distinct terms. The first term, m_z^o, is proportional to the lateral magnetic diffusion speed, represented by $c_d = 2\eta/h$ (Tyler 2005), signifying the contribution due to magnetic diffusion. The second term, g_z^o, is proportional to the leading wave speed $c_t = \sqrt{gh}$. This term accounts for the magnetic component resulting from self-induction caused by the direct forcing of the gravity wave, induced by the associated inflow of water through a control surface.

4 Discussion

The novel formulations (10)–(12) provide a direct exploration of the time-domain mechanisms behind the transient magnetic field generation. As an illustrative example, let us consider an ocean with a depth $h = 2000$ m and a Gaussian-shaped seabed displacement $H_0(x) = Ae^{-(x/\Delta)^2}$, where $A = 3$ m and $\Delta = 5000$ m. Figure 1 illustrates the time series of the free-surface elevation along with the oscillatory component of the magnetic field at a considerable distance $x = 3500$ km from the epicenter, obtained with expression (10). Notably, there is a distinct time lag between the electromagnetic (EM) signal and the tsunami. Our novel asymptotic formulae show that this time lag is due to the phase difference between the Airy function, which dominates the propagation of the tsunami, and the Scorer function, which governs the propagation of the diffusive component of the EM signal.

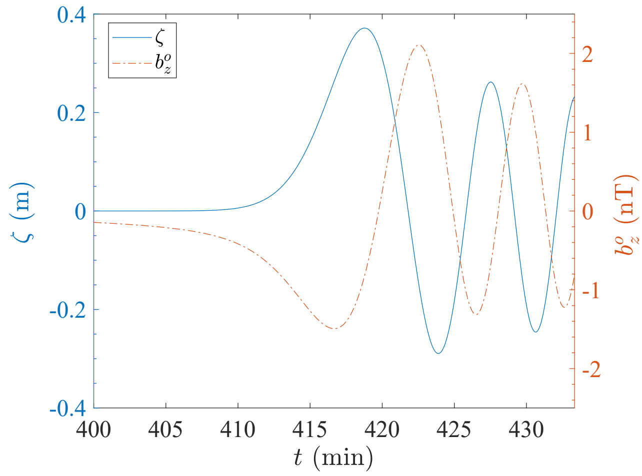

Figure 1: Time series of the free-surface elevation ζ and magnetic field b_z^o (10) at large distance $x = 3500\,\text{km}$ from the epicentre.

Importantly, figure 1 demonstrates the presence of a discernible EM signal already at $t = 400$ min. This early signal precedes the arrival of the tsunami crest at the same observation point by approximately 19 minutes. Described by the dynamo equation (5), the temporal evolution of the magnetic field is influenced by a combination of convection by the liquid's velocity and diffusion. In the example presented here, the liquid moves at a velocity close to the leading wave speed $c_t \simeq 140$ m/s. In contrast, the lateral magnetic diffusion speed is $c_d \simeq 200$ m/s. Consequently, the diffusive component of the vertical magnetic field precedes the tsunami.

In conclusion, the early detection of tsunami-generated EM signals at geomagnetic observatories has the potential to offer an advance warning on the order of tens of minutes. This advancement represents a noteworthy improvement compared to traditional tsunameter networks relying on bottom pressure sensors, which are limited to real-time detection.

References

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