# An Extended Model for Wave Impacting on a Vertical Cylinder

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## 1 Introduction

Extreme waves can generate destructive hydrodynamic loads on structures. As cylindrical structures are widely used in coastal and offshore engineering, there have been considerable research efforts on the impact of extreme waves on cylindrical structures [1, 2]. Goda et. al [3] introduced a model for calculating breaking wave impacts, which consists of a component from the Morison formula [4] and an additional term $F_s$ accounting for impact forces. The impact force is determined by utilizing the two-dimensional (2D) water slamming model proposed by von Kármán [5] and multiplying it by an assumed uniform impact area, written as

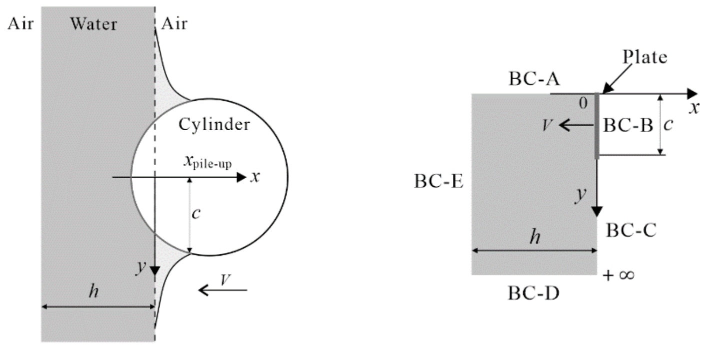
$$F_s(t) = \lambda \eta_b \cdot \rho R V^2 \cdot C_s, \quad C_s = \pi \cdot \left(1 - \frac{V}{R}t\right), \tag{1}$$

where $\eta_b$ is the wave surface elevation, $\lambda\eta_b$ represents the height of impact area, $\lambda$ is the curling factor, $\rho$ is the mass density of water, $V$ is the wave velocity, $R$ is the radius of cylinder, $C_s$ is the slamming coefficient, and $t$ is time.

As opposed to the impact force given by Eq. (1), a more accurate 2D water slamming theory would be an obvious improvement. Wagner [6] pointed out that, when a structure interacts with liquids, a bulge effect in the liquid surface exists near the solid-liquid contact area. This effect (also known as the pile-up effect) enlarges the contact area between the solid and the liquid, thus increasing the maximum slamming coefficient $C_s$ to 2π, which is twice that in von Kármán's model. This model is also called the original Wagner model (OWM). Subsequently, several improved water slamming models, including the original and the modified Logvinovich model, have been proposed for more accuracy prediction of force in water entry problems [7, 8]. For wave impacting, the most commonly used wave impact model is that proposed by Wienke and Oumeraci [9], which is based on OWM and with a polynomial stepwise function to provide a good approximation of the wetted surface. Note that, all water slamming models are directly from theory applicable to water entry problems, which are not exactly consistent with the physics of wave impacting. One of the assumptions in water entry problems is infinite water thickness, which corresponds to the width of the horizontal cross-section of surface waves. However, this width for waves is not only finite, but also relatively small near the wave crests, the region that is considered to have the most destructive impact loads. To date, the effect of this assumption has not been incorporated in wave impact problems. Korobkin [10] developed a water entry theory under a finite-water-extent with a solid boundary at the bottom, and they found that as the water thickness decreased, the pressure exerted on the structure increased. However, in the context of wave impact, the rear surface of the wave crest is a free surface boundary condition, which does not hinder the water flow. As a result, it may cause a smaller force than if it was a solid boundary. Therefore, in this study, a water slamming theory that is more suitable for wave impacts is derived by considering the finite water extent impacting with double-free water surface.

## 2 Derivation of Finite Water Extent Slamming Theory

The complexity of waves makes it challenging to construct a three-dimensional (3D) model for a cylinder subjected to wave impacts. Therefore, the strip theory is employed to vertically divide the cylinder and the wave into several sections, and then these sectional forces are computed and summed to obtain the total force acting on the entire cylinder. For one 2D wave section, it can be approximated as a semi-infinite rectangle with a finite length $h$ in the $x$-direction, and an infinite length in the $y$-direction as shown in Figure. 1. The water is considered motionless and is impacted by a cylinder of radius $R$ with constant velocity $V$. Note that the right and left boundaries of the water are both free surfaces, while the structure-fluid contact area can be characterized by the half-width of the wetted area $c$. A flat plate assumption is adopted for further simplification, whereby the cylinder is approximated by a flat plate with the same width of the wetted area and velocity, as shown in Figure. 1 (b).



(a) Original problem (b) Simplified problem

Figure. 1 Sketch of the 2D water slamming problem with simplifications

**2.1 Velocity potential**

Using the potential flow theory, the governing equation for the water domain can be expressed as

$$\nabla^2 \varphi(x, y) = 0 . \tag{2}$$

The boundary conditions in Figure. 1 (b) can be written as

$$\begin{aligned} &\partial\varphi/\partial y\big|_{y=0} = 0, \ -h \le x \le 0, \\ &\partial\varphi/\partial x\big|_{x=0} = -V, \ 0 \le y < c, \\ &\varphi\big|_{x=0} = 0, \ c < y < \infty, \\ &\varphi\big|_{y\to\infty} = 0, \ -h \le x \le 0, \\ &\varphi\big|_{x=-h} = 0, \ 0 \le y < \infty. \end{aligned} \tag{3}$$

These equations can be solved by defining the following velocity potential

$$\varphi(x, y, t, h, V) = \hat{\varphi}\left(-\frac{x}{c(t)}, \frac{y}{c(t)}, \frac{h}{c(t)}, c(t)V\right) = \hat{\varphi}\left(\hat{x}, \hat{y}, \hat{h}, \hat{V}\right), \tag{4}$$

where $\hat{\varphi}$ is $\varphi$ with the variable substitutions and can be expressed by a Fourier transform as

$$\hat{\varphi}(\hat{x}, \hat{y}) = \frac{2}{\pi}\int_0^\infty A(k)\frac{\sinh[k(\hat{h}-\hat{x})]}{\sinh(k\hat{h})}\cos(k\hat{y})dk , \tag{5}$$

where

$$A(k) = -\int_0^1 \frac{\hat{V}\tanh(\beta\xi)}{k\cosh(\beta)\sqrt{\tanh^2(\beta)-\tanh^2(\beta\xi)}}\sin(k\xi)d\xi, \ \beta = \frac{\pi}{2\hat{h}} . \tag{6}$$

Given the derived expression for the velocity potential, the next step is to calculate $c(t)$, which is usually determined by the Wagner condition [7, 11]. Howison et. al [12] provided a detailed solution with infinite-water-extent. Extending their work, the solution of the Wagner condition in the case of finite-water-extent slamming can be written in the following implicit equation

$$\frac{\pi}{2}Vt=\int_0^{\pi/2} f\left(\frac{2h}{\pi}\operatorname{asinh}\left[\sinh\left(\frac{\pi}{2h}c(t)\right)\sin\theta\right]\right)d\theta\ . \tag{7}$$

### 2.2 Pressure on the cylinder

The pressure can be calculated using the unsteady Bernoulli equation,

$$p-p_a=-\rho\left[\frac{\partial\varphi}{\partial t}+\frac{1}{2}\left(\left(\frac{\partial\varphi}{\partial x}\right)^2+\left(\frac{\partial\varphi}{\partial y}\right)^2\right)\right]+\frac{1}{2}\rho V^2, \tag{8}$$

where $p_a$ is the atmospheric pressure, and the derivatives of $\varphi$ can be converted to the derivatives of $\hat{\varphi}$ .

## 3 Results and Discussion

### 3.1 The effect *h* has on the solution

To demonstrate the results of the theoretical calculations for finite-water-extent slamming, trial calculations have been performed for cases with $R$ = 1.2 m and $V$ = 6.75 m/s, but varying $h/R$ from 0.1 to 1.6. When $h/R$ approaches infinity, the present theory can be considered to have an infinite water body and should be consistent with the von Kármán's/Wagner's model. Therefore, the results from the Wagner model corresponding to an infinite $h$ are also presented for comparison. Figure. 2 depicts the results for the velocity potential and its time/space derivatives at $t$ = 0.01$R/V$. Besides, the corresponding solution to the infinite water body slamming can be calculated based on the Wagner model. The results indicate that as $h/R$ increases, the results of the present theory gradually approach those of the Wagner model. In addition, the results for different $h/R$ have different ranges on the $y$-axis. This is because as $h/R$ increases, $c(t)$ also increases, resulting in a wider contact area between the solid and liquid, and thus a larger range of $y$.

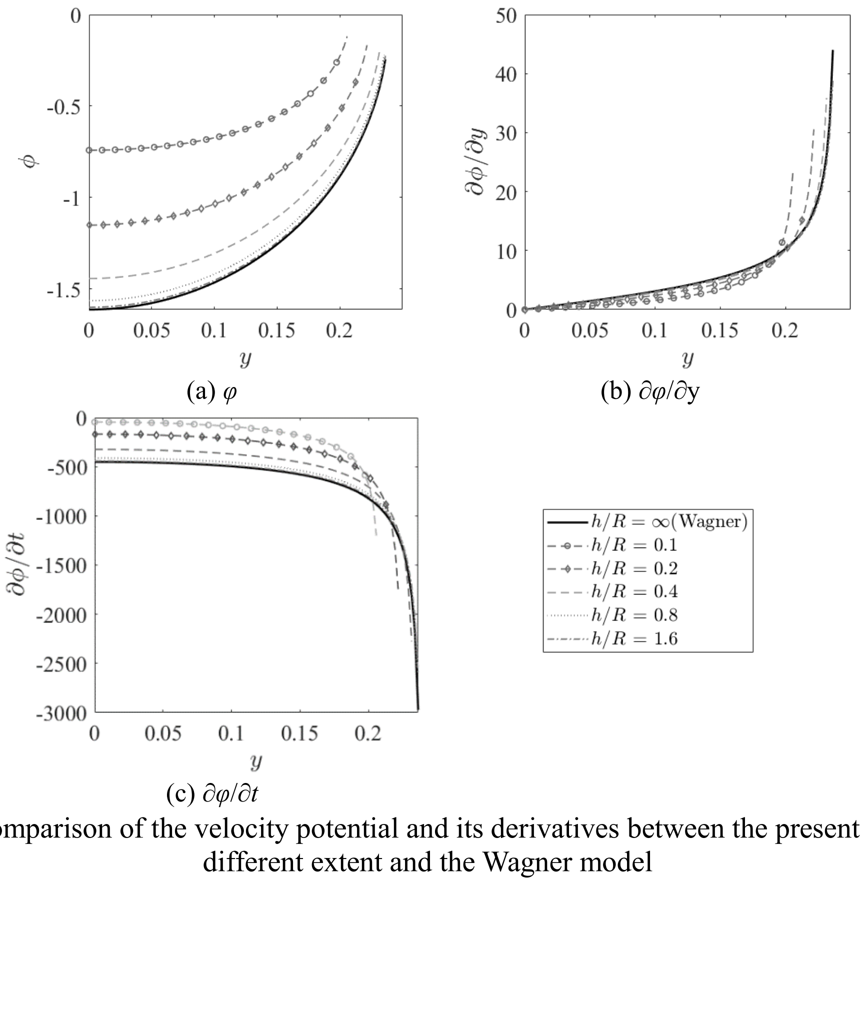
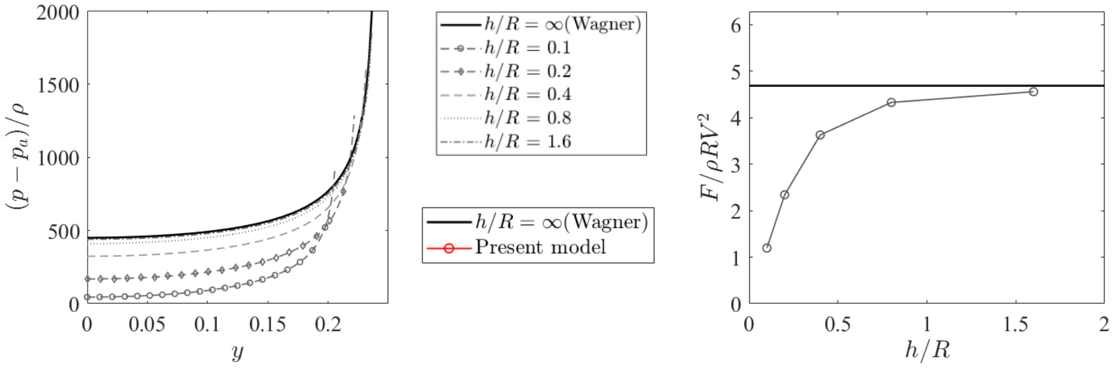



Figure. 2 Comparison of the velocity potential and its derivatives between the present model with different extent and the Wagner model

### 3.2 Comparison of pressure and sectional force

The results for the pressure and the total force obtained from the pressure integration along the cylinder are presented in Figure. 3. It can be observed that an increase in $h/R$ leads to an increase in the pressure and pressure action area on the cylinder surface. Consequently, the total force acting on the cylinder also increases and approaches that predicted by the Wagner model. Also, as the impact time approaches the initial impact moment, the influence of $h/R$ gradually decreases. This is because, at the initial impact moment, the contact area between the solid and the liquid is extremely small, and the affected fluid range is limited. Therefore, the boundary of the limited water body will not have an impact, and the impact force on the limited water body will be consistent with that on the infinite water body.

In summary, the calculation results from the finite water slamming theory show that the influence of water body width on impact forces cannot be ignored, especially in areas with particularly small water body widths (i.e., corresponding to wave crest regions). And this finite water slamming theory can be a better choice to establish the wave impact model.




(a) Pressure at $t = 0.01R/V$ (b) Sectional force at $t = 0.01R/V$

Figure. 3 Comparison of pressure and sectional force between the present model and the Wagner model

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