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Some aspects of the three-dimensional hydroelastic slamming

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1 Introduction

The solution of a conical shell impact on water within the Wagner approach was reported recently Khabakhpasheva et al. (2023). This solution is axisymmetric and fully coupled. In this approach, the hydrodynamic part of the problem is simplified. The elastic displacements of the shell elements are described within the linear theory of elastic shells of small thickness , where inertia of the shell elements in tangential direction is neglected. The flow region is locally approximated by a half-space, $z < 0$, the boundary conditions are linearised and imposed on the plane $z = 0$. In axisymmetric problems, the wetted part of the body is approximated by a circular disc, the radius of which, $a(t)$, is obtained as part of the solution. Initially, $t = 0$, the cone touches the flat water free surface, $a(0) = 0$, and then starts penetrating the water vertically at a constant speed V. The normal and tangential displacements of the elements of the conical shell are obtained using the normal modes of a circular elastic plate clamped at the outer edge and at the plate center. These modes were introduced by Scolan (2004). The solution from Khabakhpasheva et al. (2023) is based on Scolan's ideas and models.

In real problems, an elastic structure could be complex, which implies that the solution should be numerical. The solution should be coupled with the structural and hydrodynamic problems being solved at the same time, which can be achieved through iterations between a finite-element code for the structural analysis and a CFD code for the hydrodynamic loads acting over the wetted part of the elastic body. The resulting numerical model should be validated and verified for all possible regimes of impact. We suggest that investigation of a numerical model of hydroelastic slamming, its design and validation will benefit from using semi-analytical solutions of this problem, which have no hidden parameters, transparent and simple enough to understand and explain each step of the solution. These solutions can be used for validation of three-dimensional numerical models for all the interaction regimes: impulsive, dynamic, and quasi-static. This is important because the numerical parameters (mesh size and type, time step, shape and size of the computational domain, management of the free surface, and others) are strongly dependent on the type of the interaction regime. Simplified models, such as the Wagner model of hydroelastic slamming, can be also used to mimic performance of complex numerical models and improve them if it is necessary.

Semi-analytical models have also their own limitations. They should be developed to include all possible regimes of slamming. Identified difficulties with semi-analytical models are related to such physical phenomena as *ventilation, cavitation and compressibility of water*. To explain required modifications of the Wagner model, we start with the formulation of the slamming problem for a conical shell.

2 Formulation of the problem and the normal modes

Within the Wagner model, the axisymmetric problem of a conical shell impact on water is formulated in dimensionless variables, where lengthscale is the length of the cone R, the velocity scale is the impact speed of the cone V and the timescale is $R\sin\beta/V$, β is the deadrise angle of the cone. The velocity potential in the flow region $z<0$ is the solution of the following boundary problem,

$$\nabla^2\varphi = 0\,(z<0),\;\; \varphi(r,0,t)=0\,(r>a(t)),\;\; \varphi_z(r,0,t) = -1 + w_t(r,t)\,(r<a(t)), \qquad (1)$$

where the normal deflection $w(r,t)$ satisfies the following equation,

$$\mathcal{L}(w;\delta,\alpha) = \alpha\gamma p(r,0,t), \qquad (2)$$

which describes dynamics of a thin conical shell. Here $\mathcal{L}(w;\delta,\alpha)$ is a linear integro-differential equation with variable coefficients, see Khabakhpasheva et al. (2023) for details, and $p(r,0,t) = -\varphi_t(r,0,t)$ is the hydrodynamic pressure over the wetted part of the shell, $r<a(t)$. The shell is clamped at the tip of the cone, $r=0$, and at the edge, $r=1$. The dimensionless parameters in (1) and (2) are $\alpha = 12(1-\nu^2)(VR/[c_p h\sin\beta])^2$, $\gamma = \rho R/\rho_s h$, $\delta = 12\tan^2\beta R^2/h^2$, where $c_p = \sqrt{E/\rho_s}$, is the speed of a signal in the shell, ρ_s, E and ν are the density, Young module and Poisson ratio of the shell material, and ρ is the water density.

The current position of the entering shell is described by the equation $z = Z_b(r,t)$, where $Z_b(r,t) = r + w(r,t) - t$ in the dimensionless variables. The radius $a(t)$ of the contact region is governed by the Wagner condition that the elevation of the free surface at the periphery of the contact region, $r=a$, is equal to the current position of the entering body there. This condition was reduced in Korobkin & Scolan (2006) to the equation

$$\int_0^{\pi/2} \sin\theta Z_b(a\sin\theta,t)\mathrm{d}\theta = 0. \qquad (3)$$

The velocity of the contact region expansion $a'(t)$ is obtained by differentiation of (3),

$$a'(t)\int_0^{\pi/2}\sin^2\theta Z_{b,r}(a\sin\theta,t)\mathrm{d}\theta = -\int_0^{\pi/2}\sin\theta Z_{b,t}(a\sin\theta,t)\mathrm{d}\theta. \qquad (4)$$

The shell deflection is sought in the form

$$w(r,t) = \sum_{n=1}^{\infty} b_n(t)\psi_n(r), \qquad Q_n(\xi) = \int_0^{\frac{\pi}{2}} \psi_n(\xi\sin\theta)\sin\theta\mathrm{d}\theta. \qquad (5)$$

where $\psi_n(r)$ are the natural modes of a unit elastic circular plate clamped at the edge and at the centre, see Scolan (2004), and $b_n(t)$ are the coefficients to be determined.

$$\frac{\mathrm{d}\vec{b}}{\mathrm{d}t} = (\mathcal{I}+\gamma\mathcal{S}(a))^{-1}(\vec{q}-\gamma\vec{f}(a)) \qquad \frac{\mathrm{d}\vec{q}}{\mathrm{d}t} = -\frac{1}{\alpha}(\mathcal{D}+\delta\mathcal{T})\vec{b}, \quad \frac{\mathrm{d}a}{\mathrm{d}t} = \frac{1-\vec{b}_t\cdot\vec{Q}(a)}{\pi/4+\vec{b}\cdot\vec{Q}'(a)}, \quad (6)$$

where $\mathcal{D}$ is a diagonal matrix, symmetric matrices $\mathcal{S}(a)$, $\mathcal{T}$ and the vector $\vec{f}(a)$ are pre-calculated, initially $\vec{b}(0) = 0$ and $\vec{q}(0) = 0$. Once the system (6) has been integrated in time, the hydrodynamic pressure in the wetted part of the shell can be calculated using the shell equation (2) and series (5). However, the resulting series for the pressure converges very slowly. We improve the convergence by separating the singular part of the pressure,

$$p(r,0,t) = B(t)\,\frac{H(a^2-r^2)}{\sqrt{a^2-r^2}} + \sum_{n=1}^{\infty}\tilde{p}_n(t)\psi_n(r), \quad B(t) = \frac{2}{\pi}a\dot{a}\Big(1-\sum_{n=1}^{\infty}\frac{\mathrm{d}b_n}{\mathrm{d}t}Q_n(a)\Big), \quad (7)$$

where $\tilde{p}_n(t) = p_n(t) - aB(t)Q_n(a)$. The series for $B(t)$ and the series with the coefficients $\tilde{p}_n(t)$ converge quickly enough to evaluate the impact pressure accurately with a reasonable number of modes.

3 Limitations of the model of hydroelastic slamming

Note that for a rigid cone with $w = 0$, we have $Z_{b,r}(a\sin\theta, t) = 1$, $Z_{b,t}(a\sin\theta, t) = -1$ and then $a'(t) = 4/\pi$. The hydrodynamic problem (1) was derived under the condition that the derivative $a'(t)$ is finite and positive during the impact stage. Equation (4) predicts that the speed of the contact region changes its sign when the right-hand side of (4) is equal to zero, and the speed becomes unbounded when the integral on the left-hand side of (4) is equal to zero. For low speeds of impact, the integrals in (4) do not change their signs and integration of (6) can be performed without difficulties, see figure 1. For higher speeds, the deflection can be such that the shape of the cone becomes concave, which leads to decrease of the integral with $Z_{b,r}$ down to zero and to unbounded speed of the contact region expansion. In this case, the edge of the cone may contact the elevated free surface before the contact line arrives at the edge. If so, a *torus air pocket* can be trapped near the edge of the cone. For even higher speeds, the duration of the impact stage is so short that deformations of the shell are small during the impact stage but the local velocities of the deflections can be comparable with the impact speed. Then the the integral with $Z_{b,t}$ in (4) could change its sign from negative to positive, implying that the contact region starts to shrink. This effect is known as *ventilation*. Existing Wagner-type models of hydroelastic slamming cannot describe ventilation.

Another limitation of the Wagner model is that it does not describe cavitation which could be caused by impulsive deflection of the elastic body surface. The interface cavitation starts there and then, where and when the dimensionless dynamic pressure given by (7) drops below $(p_{cav} - p_{atm})\sin\beta/\rho V^2$. Here p_{cav} is the cavitation pressure, $p_{cav} \approx 2.3\times10^3$ N/m^2, which can be neglected compared with the atmospheric pressure, $p_{atm} \approx 1.013\times10^6$ N/m^2, leading to the condition of no cavitation, $p(r,0,t) > -100\sin\beta/V^2$.

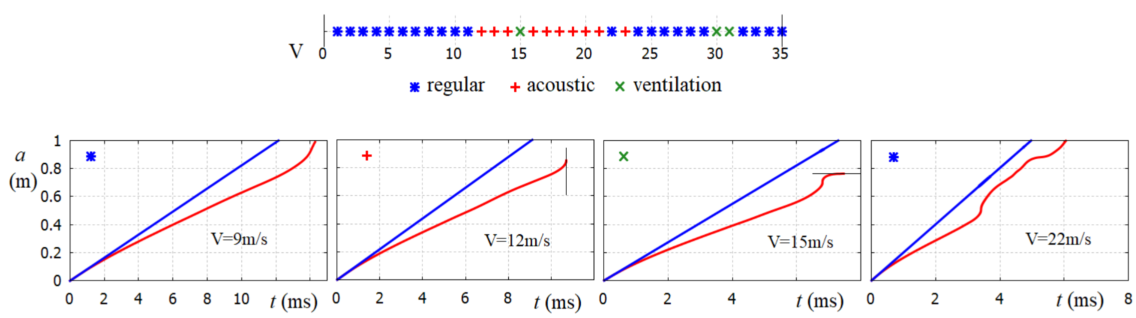

Fig. 1 The speeds of the impact where the Wagner model fully describes the impact stage (regular regime, blue) and where the Wagner model fails because the speed of the contact region expansion becomes infinite (acoustic and/or air bubble, red) or negative (ventilation, green) for the cone made of aluminum with thickness 1 cm, length 1 m and deadrise angle 8^o.

4 Coupled CFD+FE model

In principle, the coupled CFD-FE solver can compute the complex interaction of the fluid flow and the elastic structural deformations during severe hydrodynamic impact conditions. The physical complexity of the interactions, as well as the complexity of the numerical algorithms, require a rigorous and comprehensive validation. Due to its high accuracy, the semi-analytical solution which has been elaborated here, represents perfect validation case. The CFD-FE solver in consideration consists of a free-surface RANSE solver (`OpenFOAM` ® Weller et al. (1998)), a linear structural dynamic solver (mode-superposition approach) and a partitioned FSI coupling scheme Seng et al. (2014). This coupled solver was applied previously Malenica et al. (2022) to simulate axisymmetric water-entry of an linear elastic conical shell in different interaction regimes: impulsive, dynamic and quasi static. While the results were encouraging, the eigenmodes and eigenvalues of the conical shell applied in Malenica et al. (2022) were artificially modified in the CFD-FE model to comply with the assumption of the semi-analytical model of Scolan (2004). With the present improvement of the semi-analytical model this assumption is no longer needed, hence, the CFD-FE model can be validated as-is and the simulation conditions represent more realistically the actual hydroelastic behaviour of the conical shell. The difficulties in CFD to include additional physics, such as ventilation and cavitation, are as critically important as for the Wagner model. While the numerical algorithm allows these effects to be included in the simulations, the complexity of the phenomena makes it hard to obtain a strong evidence for the validity of the results.

Several test cases covering all different interaction regimes are under consideration and the numerical results will be presented at the Workshop.

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