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The effect of variation of cross-sectional geometry on the performance of a top-hinged wave energy converter

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1 Introduction

Geometry optimisation of a wave energy converter (WEC) is an excellent way to improve performance of the WEC, since wave-structure interaction and the resulting forces depend strongly on the geometry (Garcia-Teruel & Forehand 2021). In a recent study, a geometry optimisation of a top-hinged WEC was performed, wherein the objective functions were to (i) maximise power and (ii) minimise the power take-off (PTO) force, forming a multi-objective optimisation. The latter objective function was introduced because the PTO equipment can incur up to 50% of the total capital expenditure of the WEC (Bedard et al. 2004). In this previous study, a uniform cross-sectional area for the top-hinged WEC in the $x - z$ plane was considered, as shown in figure 1a. Results were shown to be consistent with relevant far-field theory of radiating waves. It was shown that by introducing minimisation of the PTO force as an objective function, we could significantly lower the PTO force without significantly reducing the extractable power.

In the present study, we look to generalise the geometry optimisation further by relaxing the requirement of uniform cross-sectional area. The shape of the $x - y$ plane geometry is changed, and the performance of the WEC is studied (*i.e.*, the extractable power and PTO reaction force).

The methodology is outlined in Figure 1. The WEC is a top-hinged WEC, whereby the main WEC absorber is attached to a hinge point (O) via a rigid arm and thus restricted to pitch motion only. In the previous study, we defined curves c_1 and c_2 to describe the $x-z$ plane curves defining the front and rear face of a top-hinged WEC with uniform cross section. We performed a multi-objective optimisation to find the optimal geometries (that is, the parametric expressions for the curves c_1 and c_2, as well as parameters r_1 and r_2, which define the horizontal lengths, as shown in Figure 1a), which (i) maximise power, and (ii) minimise PTO force. Extending this work, here we select five representative cross-sectional areas from the previous optimisation's Pareto Front of optimal solutions, distinguished by colour in Figure 1c. In this analysis, we define curves c_3 and c_4 to represent a change in the cross-sectional area along the width of the device. We look at the effect of changing parameters f_2 and g_2, which are the second-order coefficients of the polynomial basis functions defining c_3 and c_4, respectively, on the power and force.

2 Theory

In this study, we assume the wave amplitude to be small and the fluid to be ideal, allowing linear potential flow theory to be used. We assume that the incident wave has a given

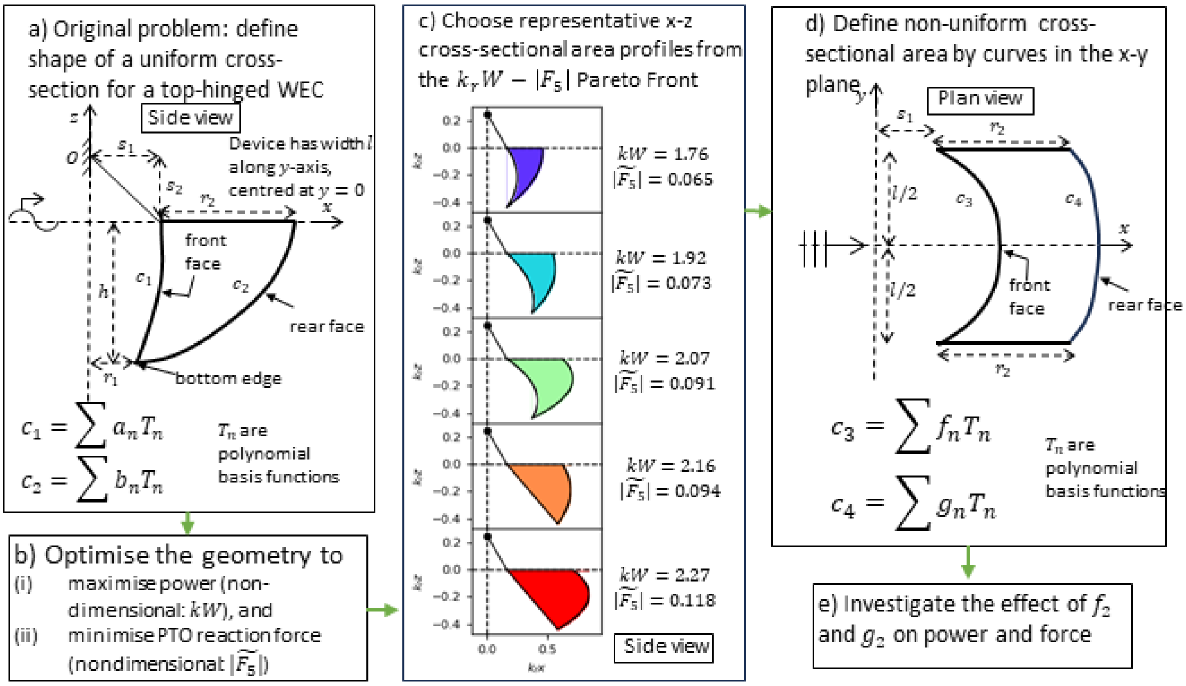

Figure 1: Flow chart of the procedure to define the geometry.

frequency ω and angle $\theta = 0$, there is a constant water depth, $kH = 5.34$, and that the PTO force can be modelled as linear damper, $F_5 = \beta_{55}\dot{\xi}_5$, where β_{55} is the PTO damping coefficient and ξ_5 is the body motion in pitch. Since the WEC is restricted to motion in pitch, the equation of motion is

$$(I_{55} + A_{55})\ddot{\xi}_5 + (\beta_{55} + B_{55})\dot{\xi}_5 + C_{55}\xi_5 = X_5, \tag{1}$$

where I_{55} is pitch moment of inertia, A_{55} is pitch added mass, B_{55} is pitch radiation damping, C_{55} is pitch hydrostatic coefficient, and X_5 is pitch excitation force.

Extractable power, averaged over one period, for this WEC is $P = \frac{1}{2}\beta_{55}\omega^2|\xi_5|^2$. To maximise P, we set $\beta_{55} = B_{55}$ and $C_{55} - \omega^2(I_{55} + A_{55}) = 0$, which result in optimal power, $P^{\text{opt}} = \frac{|X_5|^2}{8B_{55}}$, occurring when $|\xi_5^{\text{opt}}|/A = |X_5|/(4\omega B_{55})$, and thus $F_5^{\text{opt}} = \frac{|X_5|}{2}$. It is convenient to define capture width W to be extractable power over incident wave power per unit crest length, $P_I = \rho g A^2 V_g/2$. Additionally, we nondimensionalise W with wavenumber k.

It is well-known (Newman 1976), (Mei 1976) that radiation damping can be expressed in terms of the far-field amplitude of the wave produced by the oscillating body in otherwise still water, using Kochin functions $H_j(\vartheta)$. For power, in terms of far-field behavior, we get an expression for optimal capture width:

$$kW^{\text{opt}} = \pi\frac{|H_5(\pi)|^2}{\int_0^{2\pi}|H_5(\vartheta)|^2 d\vartheta}. \tag{2}$$

Furthermore, PTO force in terms of far-field behavior is:

$$|F_5|^{\text{opt}} = \frac{\rho\omega V_g}{k}|H_5(\pi)|. \tag{3}$$

We nondimensionalise force as $|\tilde{F_5}| = F_5 k^3/(\rho\omega^2 l)$, where l is the width of the WEC.

From these equations, we can see that to maximise power, we must maximise the wave in the $x = -\infty$ direction, when the WEC is forced to move in otherwise calm water, and minimise waves in all other directions. To minimise PTO force, we want to minimise the wave in the $x = -\infty$ direction, when the WEC is forced to move in otherwise calm water. Therefore, we have competing goals of minimising and maximising the wave in the $x = -\infty$ direction, and an unchallenged goal of minimising wave in all other directions. We now explore how the shape of the WEC impacts these objectives.

3 Results and discussion

To examine the effect of the non-uniformity of the cross-sectional area on the power and force, we change the coefficients f_2 and g_2. The coefficient f_2 governs the shape of the front face of the device: as shown in Figure 2a, a positive f_2 corresponds to a more convex front face, and a negative value of f_2 corresponds to a more concave front face. g_2 determines the shape of the rear face of the device: as shown in Figure 2d, a positive value of g_2 corresponds to a more concave rear face, and a negative value of g_2 corresponds to a more convex rear face.

Taking five shapes from the original optimisation with uniform cross sectional area (*i.e.* $f_2 = g_2 = 0$), we change f_2 and g_2 and study the impacts. The five selected shapes (shown in Figure 1c) represent the spread of power and force values on the Pareto Front resulting from the original optimisation. The colours are kept consistent in Figure 2b,c,e and f, which show how the power (b and e) and force (c and f) depend on f_2 and g_2.

From Figure 2b and c, we can see that power is increased with a more convex front face (a more positive f_2 value) for all five shapes, whereas force decreases with both convex and concave front faces for all five shapes. Using far-field theory, we can infer that both convex and concave front faces reduce the wave in the $x = -\infty$ direction compared to a flat front face (which consequently reduces the force, as shown in equation 3). However, a more convex front face reduces waves in a wider directional spread (and thus reduces the denominator in equation 2 and increases power). This suggests that a more convex front face will both increase power and reduce PTO force - two desirable traits for a WEC.

From Figure 2e, we can see that the g_2 value which maximises power is different for the five shapes: for the highest-power highest-force (red) shape, the power is maximum when $g_2 = 0$. However, for the lower-power lower-force shapes, the power is maximised when the rear face is more concave. These results are surprising: intuition from far field theory suggests that a more convex rear face would be preferable as it would minimise radiated waves over a larger directional spread.

From Figure 2f, we can see that for high-power high-force shapes (red, orange and green), the force increases with both convex and concave rear faces, with the force increasing considerably for a convex rear face. For the low-power low-force shapes (blue and indigo) the force decreases when the rear face is more concave. These results are also surprising — from far-field theory the force does not depend on waves radiated in any direction other than $x = -\infty$. One possible explanation for more convex rear faces resulting in higher power (for the low-power, low-force shapes) and higher force is that the WEC structure is becoming larger (*i.e.* the WEC volume has increased as the rear face becomes more convex). However, more work is required to explain the variation in the optimal values of g_2 for different shapes to maximise power, and the increase in force for concave rear faces (negative g_2).

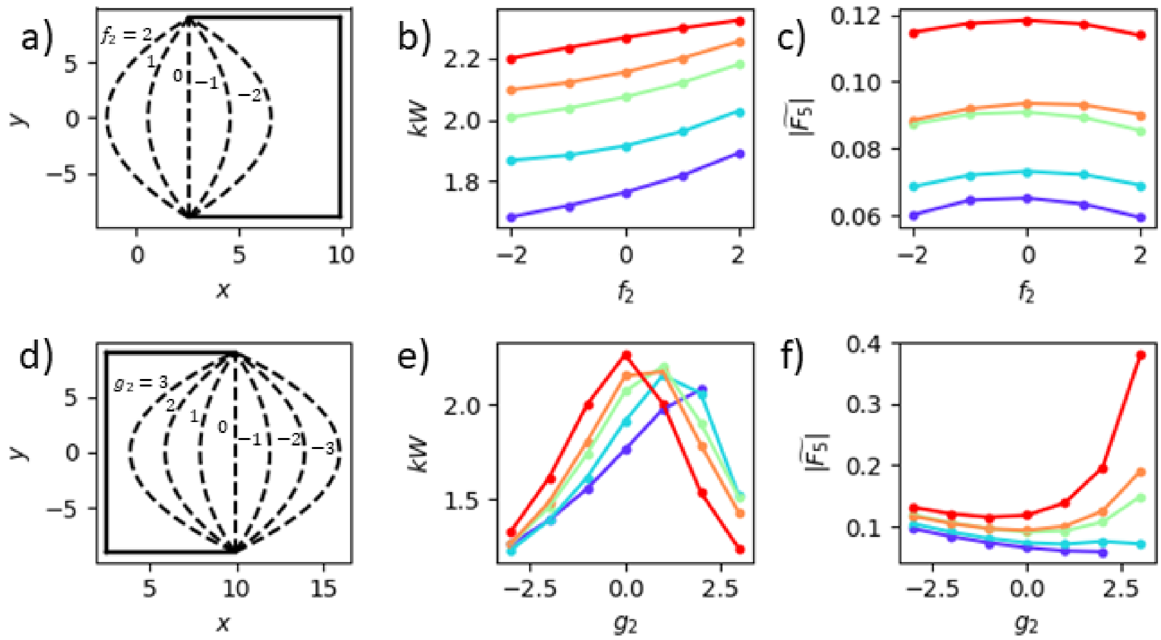

Figure 2: a) The profile, in the $x-y$ plane, of different values of f_2, b) Non-dimensionalised power (kW) vs. f_2, c) Non-dimensionalised PTO force ($|\tilde{F}_5|$) vs. f_2, d) The profile, in the $x-y$ plane, of different values of g_2, e) Non-dimensionalised power (kW) vs. g_2, f) Non-dimensionalised PTO force ($|\tilde{F}_5|$) vs. g_2. In b, c, e, and f, the colour correspond to the respective shape in Figure 1c.

4 Conclusion

This study looks at the effect of varying cross-sectional geometry of a top-hinged wave energy converter on its power and reaction force. Far-field theory is used to explain physical insights for the resulting effects of the changing front and rear face. This theory proves reliable in explaining the results for the best front faces, but there are some surprising results for the optimal rear faces. Further work will investigate the reasons why convex rear faces do not generally appear more preferable.

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