A new pressure drop model for porous plate in waves with consideration of boundary layer frictional effect

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Highlights

- Experiments show that existing pressure drop models are not uniformly valid for wave steepness.
- We develop a new pressure drop model with consideration of frictional effects of porous wall boundary layer.
- The present pressure drop model is validated with a series of flume tests.

1 Introduction

Porous or perforated structures have a number of desirable features such as wave energy dissipation and reducing wave loads on marine structures. Different types of porous structures have been designed in recent years, such as perforated breakwater, anti-rolling tanks with porous baffles, etc. However, it is still challenging to accurately predict hydrodynamic performance of porous structures owing to complicated wave energy dissipation mechanisms under different conditions.

The boundary condition imposed on the porous structures is a key for the prediction accuracy of theoretical models on hydrodynamic characteristics. In theoretical models, this energy dissipation boundary condition has been simplified to be a pressure drop model. Two types of pressure drop models have been proposed until now. One refers to a linear pressure drop condition (LPDC), which was proposed by Sollitt and Cross (1972)[1] and further developed by Yu (1995)[2]. It is easy to apply LPDC in theoretical analysis, however, unknown artificial/empirical coefficients in LPDC need to be determined either by experiments or by time-consuming Navier-Stokes-equations-based CFD simulations. The other type is a quadratic pressure drop condition (QPDC). Currently, two kinds of QPDC are used, denoted by QPDC I and II. QPDC I was proposed by Molin (1993)[3] in which for a fixed vertical porous plate, the pressure drop Δp can be written as

$$\Delta p = p_- - p_+ = \frac{1-\tau}{2\gamma\tau^2}\rho u|u|, \tag{1}$$

where u is the horizontal velocity across the plate; τ is the porosity; ρ is the fluid density; γ is an empirical parameter taken as 0.5 in Molin and Remy (2013)[4]. QPDC II was developed by Mei et al. (1974)[5] with the pressure drop expressed by

$$\Delta p = \rho\left(\frac{F}{2}u|u| + L\frac{\partial u}{\partial t}\right) \text{ with } F = \left[\tau^{-1}\left(0.6+0.4\tau^2\right)^{-1} - 1\right]^2 \text{ and } L = \frac{2b}{\pi}\ln\frac{1}{2}\left(\tan\frac{\pi}{4}\tau + \cot\frac{\pi}{4}\tau\right), \tag{2}$$

where F is the head loss coefficient, L is the effective orifice length, and $2b$ is the width of two adjacent thin plates.

The QPDCs were developed based on the assumption that the energy dissipation is dominantly caused by the flow viscous drag effect, due to flow separation and vortex shedding, on the porous plate. This was verified by experiments for wave conditions where the viscous drag dominates the energy dissipation. However, this explanation is not applicable for conditions with relatively low wave steepness where flow separation and vortex shedding do not apparently exist near the porous plate. Indeed, our computations show that the analysis with the existing QPDCs predict the wrong results when the wave steepness is relatively low. With low wave steepness, energy dissipation through a porous plate is dominated by friction on the plate, which is a linear damping effect that was not considered in the existing QPDCs. Therefore, in this study, we present a new pressure drop model for porous structures, which is uniformly valid for both mild and steep waves. We examine the performance of the new pressure drop model by comparing the wave reflection coefficient of the porous plate computed with various existing quadratic pressure drop models with flume experimental tests for a wide range of wave steepness.

2 Derivation of a new pressure drop model

We consider the energy dissipation within porous wall boundary layers when waves pass through a vertical porous plate, as shown in Fig. 1. Assume the thickness of boundary layer on porous structures being δ, and $k\delta \ll 1$, where k is wavenumber, then the total averaged wave energy without any loss in the boundary layer in a wave period can be calculated by

$$\bar{E}_{\text{total}} = 2\bar{E}_{\text{kinetic}} = \frac{2}{T}\int_t^{t+T} \mathrm{d}t \int_{-h}^{\eta\approx 0} (1-\tau)\,\mathrm{d}z \int_{-\delta}^{\delta} \frac{1}{2}\rho\left(u^2(x,z,t) + w^2(x,z,t)\right)\mathrm{d}x, \tag{3}$$

where T and h denote wave period and water depth, respectively, and $u(x, z, t)$ and $w(x, z, t)$ represent the horizontal and vertical velocity, respectively. In Eq. (3), the factor $1-\tau$ is introduced by considering that the boundary layer only exists on the solid wall. We also assume that the mean potential energy is equal to the mean kinetic energy $\bar{E}_{\text{kinetic}}$. Upon considering the effect of porous wall boundary layer, the vertical velocity will be modified as $w'(x,z,t)$, the total wave energy can thus be calculated by

$$\bar{E}'_{\text{total}} = 2\bar{E}'_{\text{kinetic}} = \frac{2}{T}\int_t^{t+T} \mathrm{d}t \int_{-h}^{\eta\approx 0} (1-\tau)\,\mathrm{d}z \int_{-\delta}^{\delta} \frac{1}{2}\rho\left(u^2(x,z,t) + w'^2(x,z,t)\right)\mathrm{d}x. \tag{4}$$

By further assuming that the horizontal velocity is not affected by the presence of the porous wall boundary layer, we obtain the averaging energy loss given by

$$\Delta\bar{E} = \bar{E}_{\text{total}} - \bar{E}'_{\text{total}} = \frac{2}{T}\int_t^{t+T} \mathrm{d}t \int_{-h}^{\eta\approx 0} (1-\tau)\,\mathrm{d}z \int_{-\delta}^{\delta} \frac{1}{2}\rho\left(w^2(x,z,t) - w'^2(x,z,t)\right)\mathrm{d}x. \tag{5}$$

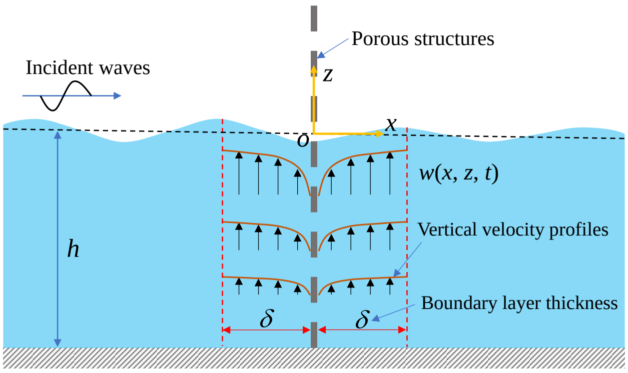

Fig. 1 Schematic diagram of vertical velocity profiles in the porous wall boundary layer

The linear pressure drop condition due to the friction effect in boundary layers can be simply expressed as $\Delta p = \beta\rho u$, where p denotes the dynamic pressure and the coefficient β can be determined approximately by the following derivation. The energy loss within the porous wall boundary layer can also be calculated alternatively as the work done by pressure. Then we have the following expression

$$C_g \Delta\bar{E}_{\text{pressure}} = \frac{1}{T}\int_{-h}^{\eta\approx 0} \mathrm{d}z \int_t^{t+T} \mathrm{d}t \underbrace{\int_{-\delta}^{\delta} \Delta p \cdot w(x,z,t)\,\mathrm{d}x}_{\text{Vertical energy flux per unit width}}, \tag{6}$$

where $\Delta\bar{E}_{\text{pressure}}$ is the energy dissipation obtained from the pressure loss, and C_g is the group velocity of wave. It is assumed that within the boundary layer the horizontal velocity is approximately equal to the vertical velocity. Based on $\Delta\bar{E} = \Delta\bar{E}_{\text{pressure}}$ and using Eq. (5) and Eq. (6), we have

$$\beta = C_g \frac{2\int_t^{t+T} \mathrm{d}t \int_{-h}^{\eta\approx 0} (1-\tau)\,\mathrm{d}z \int_{-\delta}^{\delta} \frac{1}{2}\rho\left(w^2(x,z,t) - w'^2(x,z,t)\right)\mathrm{d}x}{\int_t^{t+T} \mathrm{d}t \int_{-h}^{\eta\approx 0} \mathrm{d}z \int_{-\delta}^{\delta} \frac{1}{2}\rho w^2(x,z,t)\,\mathrm{d}x}. \tag{7}$$

Suppose that the velocity profiles near the porous boundary layer can be described by

$$w'(x,z,t) = \left[2\frac{|x|}{\delta} - \left(\frac{|x|}{\delta}\right)^2\right] w(x,z,t) \quad \text{for } |x| \le \delta, \tag{8}$$

in which $w(x, z, t)$ is the vertical velocity without considering the effect of porous wall boundary layer. The vertical velocity profile in Eq. (8) is close to the Blasius velocity profiles for a laminar boundary layer. At small wave steepness the Reynolds number is small, and the laminar boundary layer applies. Substitution of Eq. (8) into Eq. (7) results in $\beta = 14/15(1-\tau)C_g$.

Combining the above linear pressure drop model with the quadratic model in Molin (1993)[3], we have the following unified pressure drop model

$$\Delta p = \frac{14}{15}(1-\tau)C_g \rho u + \frac{1-\tau}{2\gamma\tau^2}\rho u|u|, \tag{9}$$

where the empirical discharge coefficient γ in the quadratic (velocity) term is in the range [0.5, 1.0]. We take the value of γ being 1.0 in Eq. (9). Note that no empirical coefficient appears in the linear velocity term.

3 Applied in waves interaction with a porous plate

We now examine the present pressure drop model in Eq. (9) in the problem of water waves through a porous plate. We take a Cartesian coordinate system (x, z) with origin O at the intersection of the porous plate and the mean water surface, and the direction of incident waves points to the positive x-axis. We further assume that the fluid is ideal fluid and waves propagate with small amplitudes. Then there exists a velocity potential Φ, which satisfies the Laplace equation within the fluid region, the linearized free surface condition, and no flow condition on the flat bottom. Based on the linear potential theory, we can express $\Phi(x, z, t)$ [6]as

$$\Phi(x,z,t) = \mathrm{Re}\left\{-\frac{\mathrm{i}HgN_0}{2\omega\cosh kh}\phi(x,z)\mathrm{e}^{-\mathrm{i}\omega t}\right\}, \tag{10}$$

in which H is the wave height, ϕ(x, t) is the spatial velocity potential, t represents time, $\mathrm{Re}\{\cdot\}$ denotes the real part of the argument, $\mathrm{i} = \sqrt{-1}$, and g is the gravity acceleration. The spatial potential $\phi(x, z)$ can be expanded in the form

$$\phi_-(x,z) = \left(\mathrm{e}^{\mathrm{i}kx} + R\mathrm{e}^{-\mathrm{i}kx}\right)\psi_0(z) + \sum_{n=1}^{\infty} A_n \mathrm{e}^{k_n x}\psi_n(z) \text{ for } x<0;\ \phi_+(x,z) = T\mathrm{e}^{\mathrm{i}kx}\psi_0(z) + \sum_{n=1}^{\infty} B_n \mathrm{e}^{-k_n x}\psi_n(z) \text{ for } x>0, \tag{11}$$

where R and T are complex constants whose modulus denote reflection and transmission coefficients, A_n and B_n represent the coefficients for the nth evanescent mode, and for n = 0, 1, 2, ⋯,

$$\psi_n(z) = N_n^{-1}\cos k_n(z+h),\ N_n^2 = \frac{1}{2}\left(1 + \frac{\sin 2k_n h}{2k_n h}\right), \quad \text{with } k_0 = -\mathrm{i}k,\ \omega^2 = gk\tanh kh \text{ and } \omega^2 + gk_n \tan k_n h = 0.$$

We use the boundary conditions on the porous plate, i.e., continuity of the normal velocity and the QPDC. By solving a nonlinear matrix system, we can determine the coefficients A_n and B_n, and the wave reflection coefficient.

4 Experiments

We perform a series of experiments in the wave flume at Southern University of Science and Technology. The wave flume is 20 m long, 0.8 m wide and 1.2 m deep; water depth used in the present work was set as 0.5 m. Regular waves were generated by a piston-type wavemaker, and reflection waves were absorbed by a sloping beach. A porous plate with 2 cm thickness was placed at 7.5 m away from the wavemaker and spanned the width of the wave flume. Two plate porosities ($\tau = 0.2$ and $\tau = 0.3$) were used in the present experiments. To obtain wave reflections and transmissions, we used six calibrated wave gauges to measure wave elevations. Four wave gauges were placed on the windward side of the porous plate with the position x = -3.30 m, -2.75 m, -2.30 m, and -1.10 m. Two wave gauges were located on the leeward side with x = +2.0 m and +3.0 m. Wave conditions with varying wave steepness and frequencies were tested.

5 Results, discussion and conclusions

To examine the performance of different pressure drop models, in our experiments, the wave height increases from 4.0 mm to 90.0 mm. Wave steepness kA can vary from 0.008 to 0.187. For wave heights from 4.0 mm to 18.0 mm, we also use cameras to capture the small free surface elevations. The incident wave period is 1.0 s. We present the wave reflection coefficient K_r in Fig. 2. It can be seen that the value of K_r gradually increases with increasing wave steepness in the experiments for $\tau = 0.2$ and 0.3. Moreover, the value of K_r does not turn to zero at small wave steepness as indicated by the experimental data. Specially, for τ being 0.2 and 0.3, the value of K_r is equal to about 0.2 and 0.15 when kA approaches zero, respectively. However, the theoretical results based on both QPDC I and II show a rapid change of K_r for kA within the range [0.0, 0.05]. Furthermore, the value of K_r tends to zero for small wave steepness, which is inconsistent with the experimental data. The prediction of K_r with the newly proposed QPDC match the experimental data in the whole range of wave steepness used in this work. This indicates that the pressure drop model with the consideration of the effect of porous wall boundary layers is uniformly suitable for both mild and steep waves. More detailed discussion and interpretation of the results will be presented at the workshop.

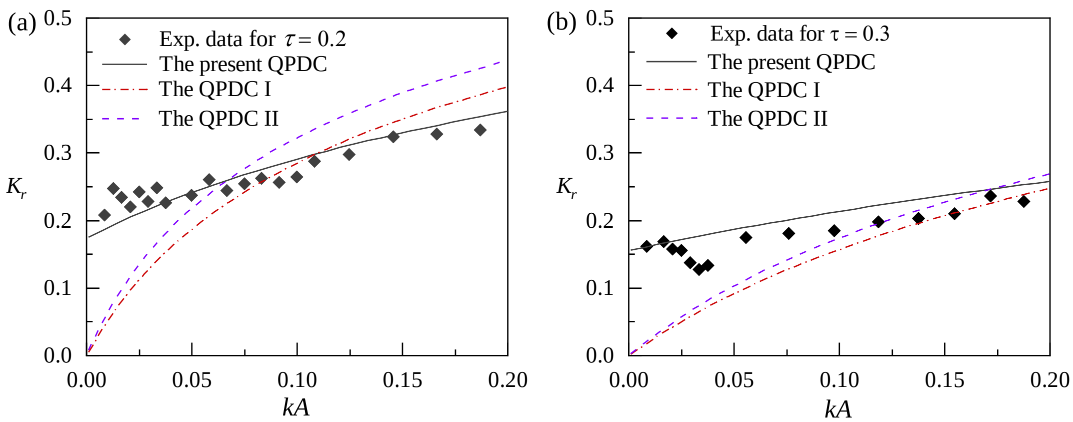

Fig. 2 Comparison of wave reflection coefficients versus kA with different QPDCs for (a) $\tau = 0.2$ and (b) $\tau = 0.3$

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